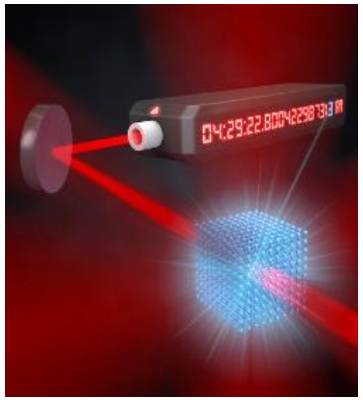
Reshaping Metrology with Quantum Information Processing

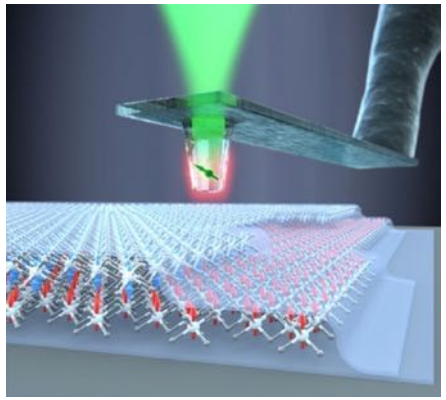
Sisi Zhou

PERIMETER INSTITUTE

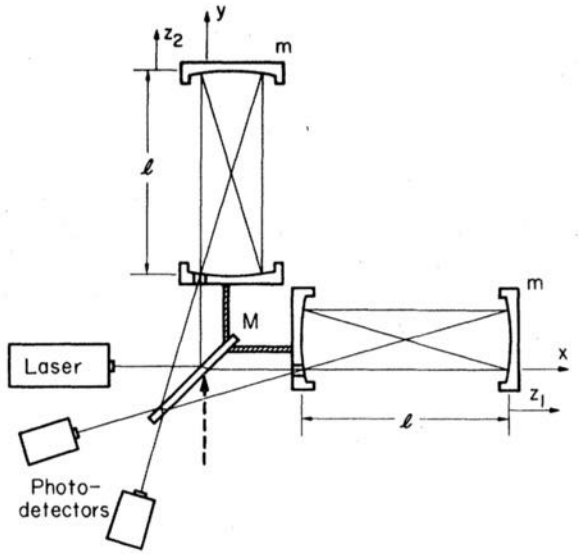
Quantum metrology is the science of estimation in quantum systems.

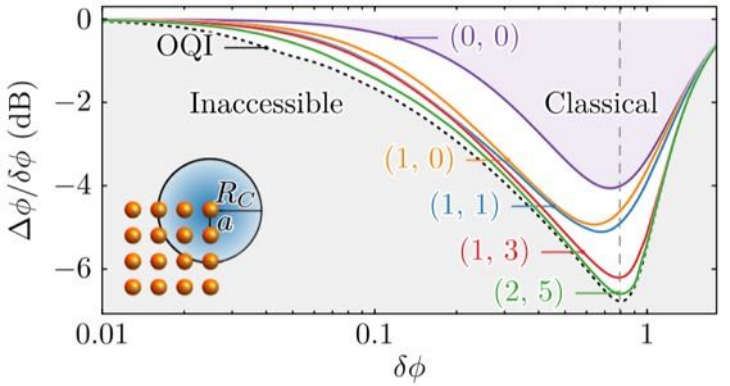


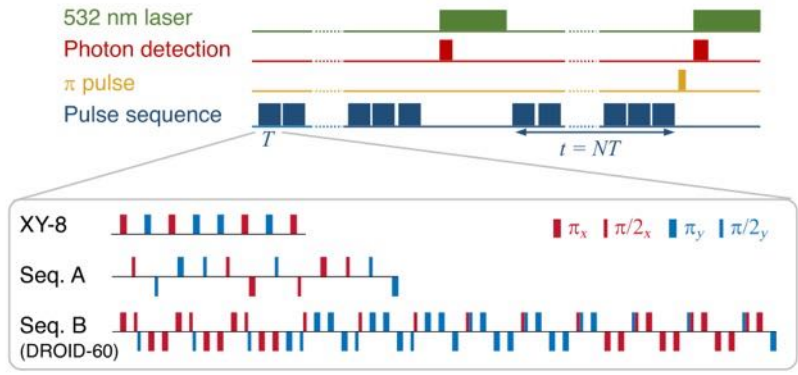




...







Optical interferometry

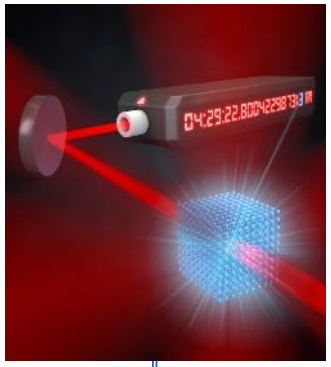
Atomic clock

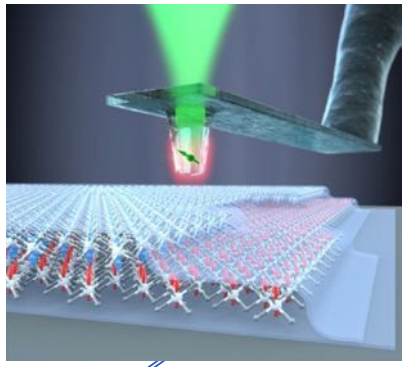
Nitrogen-vacancy centers

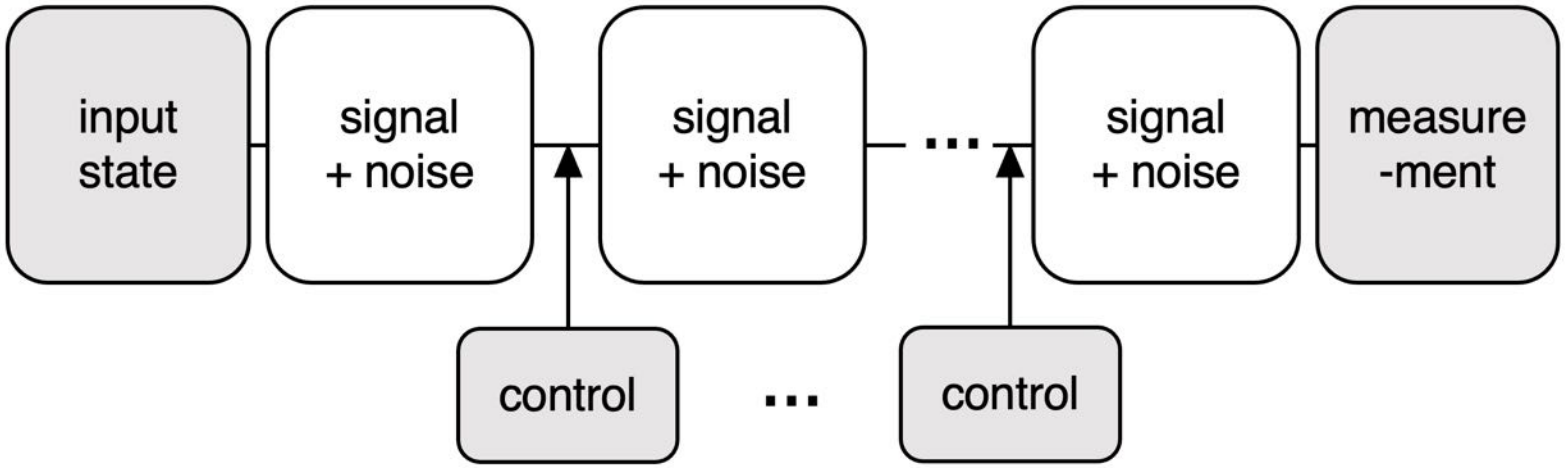
Images from: https://www.ligo.caltech.edu/news/ligo20191015, https://www.photonicsviews.com/atomic-clock-with-a-3d-optical-lattice/ https://physicsworld.com/a/diamond-quantum-microscope-images-nanoscale-features-in-2d-magnets/, PhysRevD.23.1693, PhysRevX.11.041045, PhysRevX.10.031003

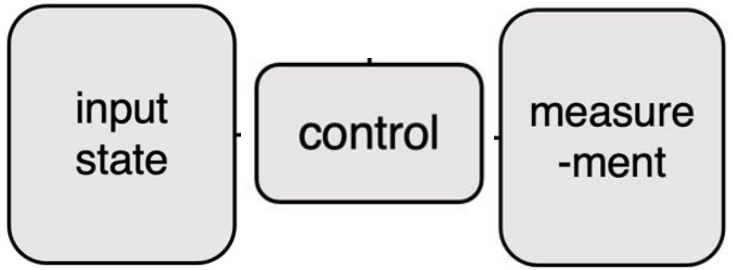
Quantum metrology enhanced by quantum information processing?

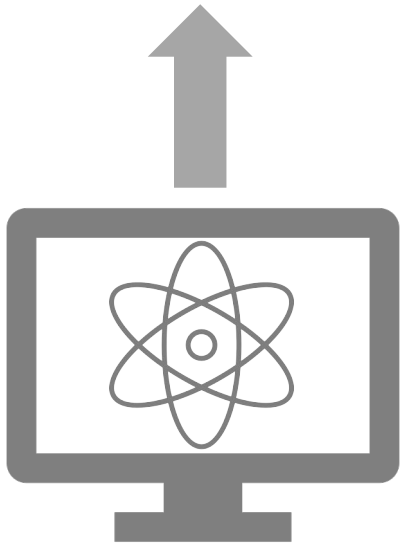








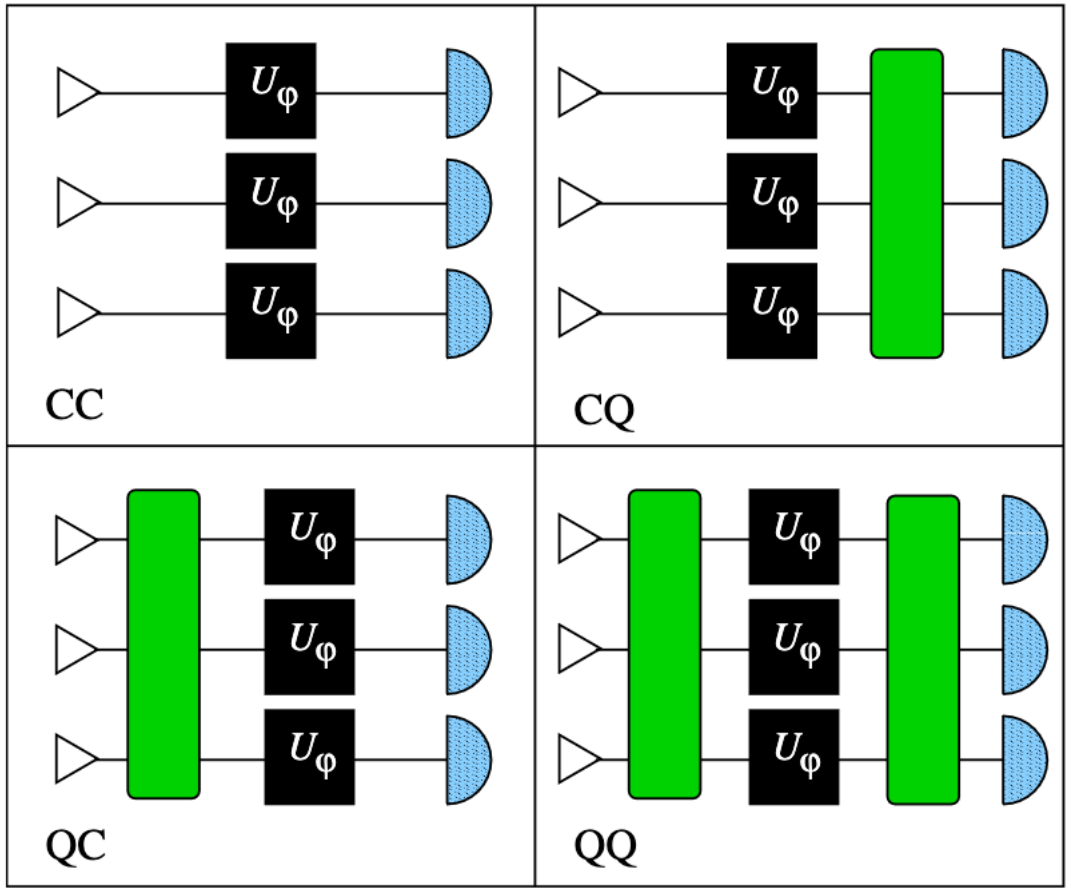




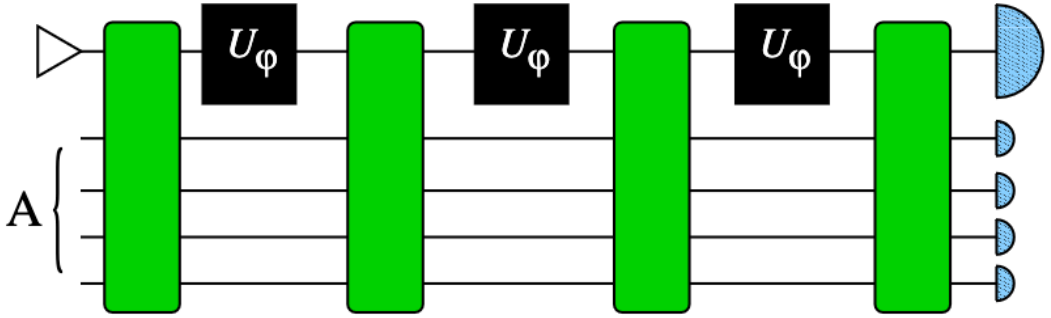
Quantum Computer

Quantum Metrology

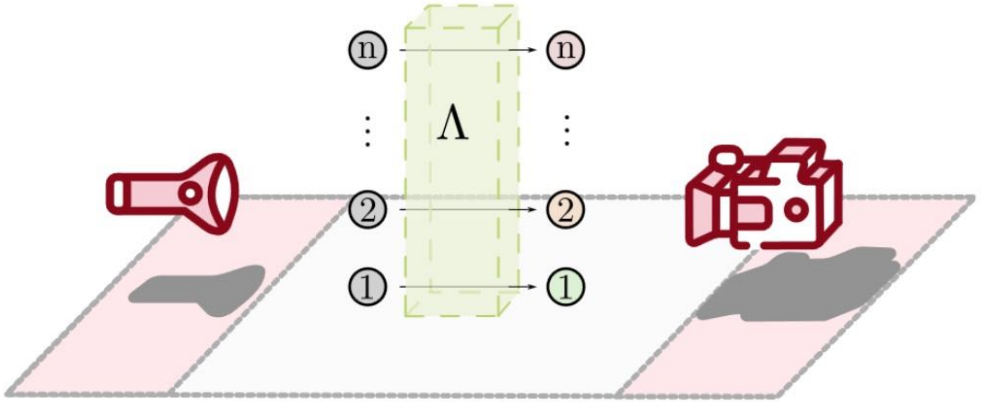
Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]
[1]*NEST-CNR-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126, Pisa, Italy*
[2]*MIT, Research Laboratory of Electronics and Department of Mechanical Engineering, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*
[3]*QUIT–Quantum Information Theory Group, Dipartimento di Fisica "A. Volta" Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*
(Received 26 September 2005; published 3 January 2006)

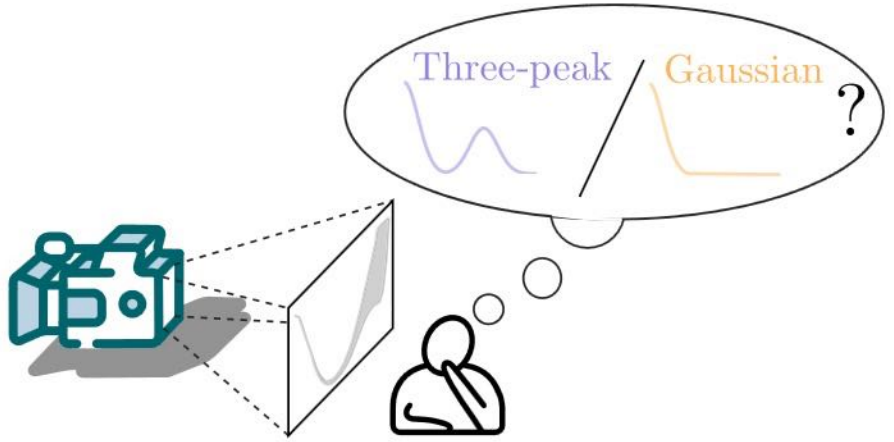


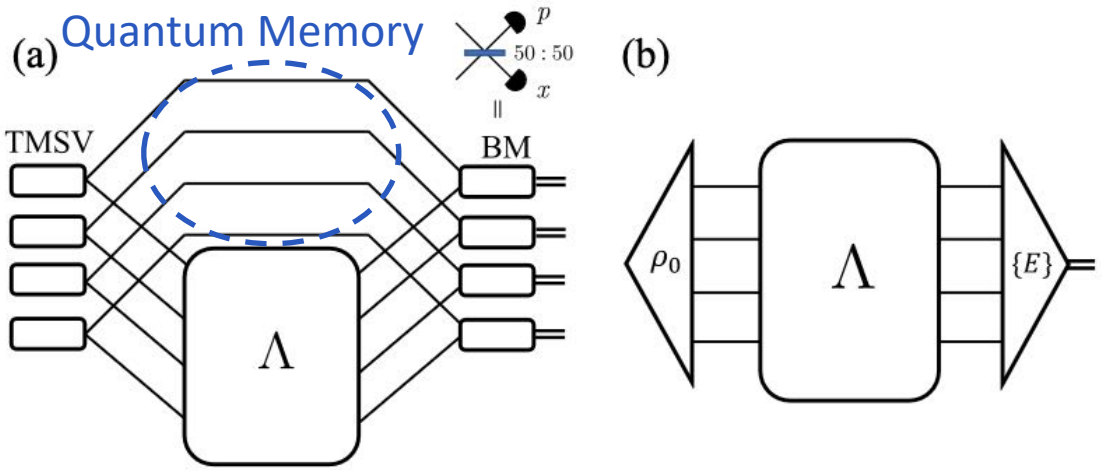
"Quantum Strategy" vs "Classical Strategy"



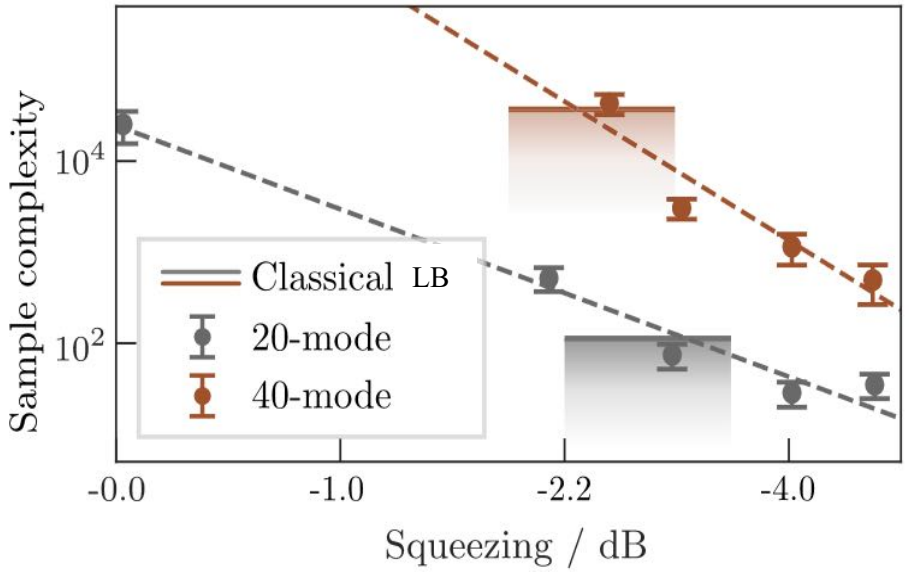
Random Displacement Sensing on Scalable Photonic Platform





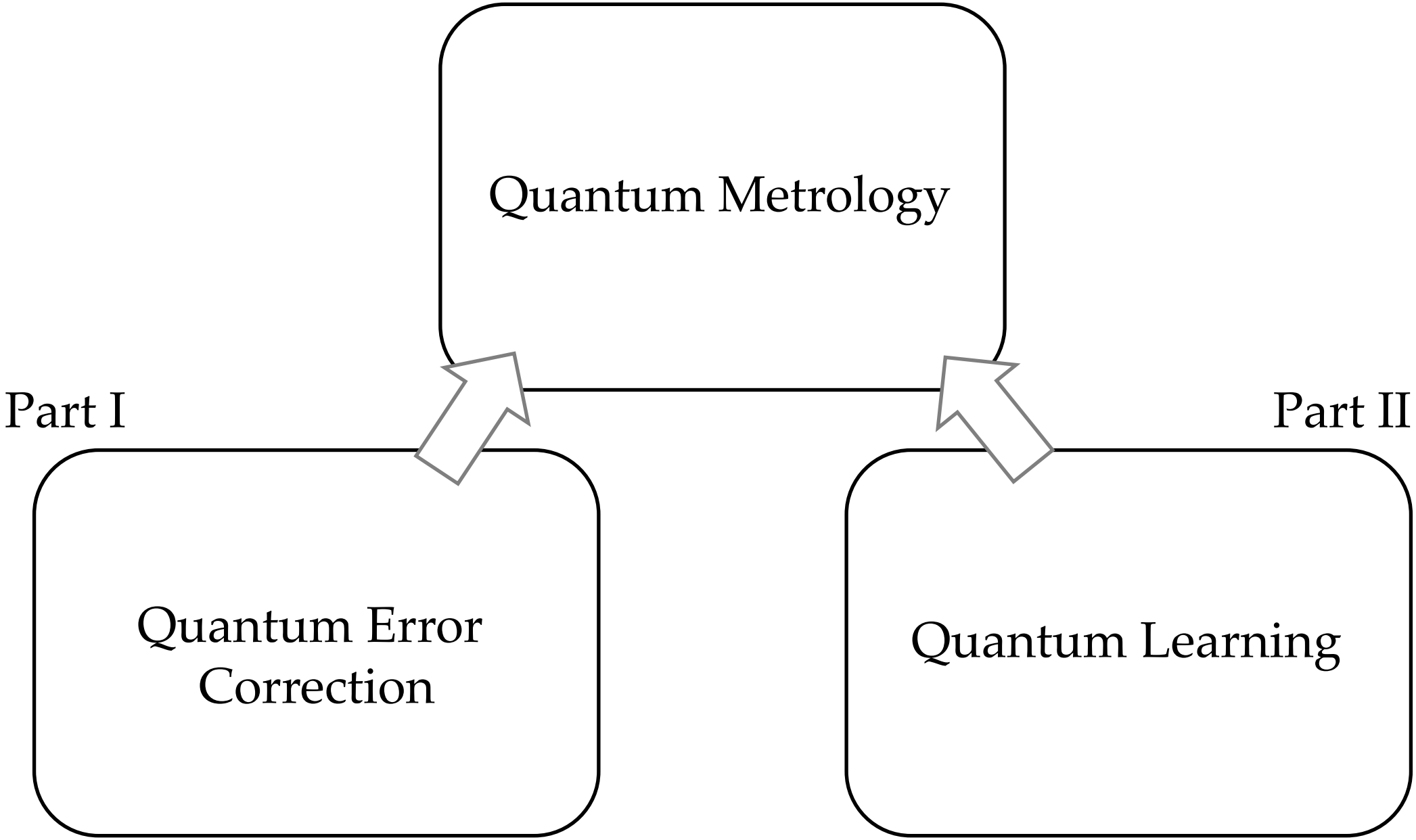


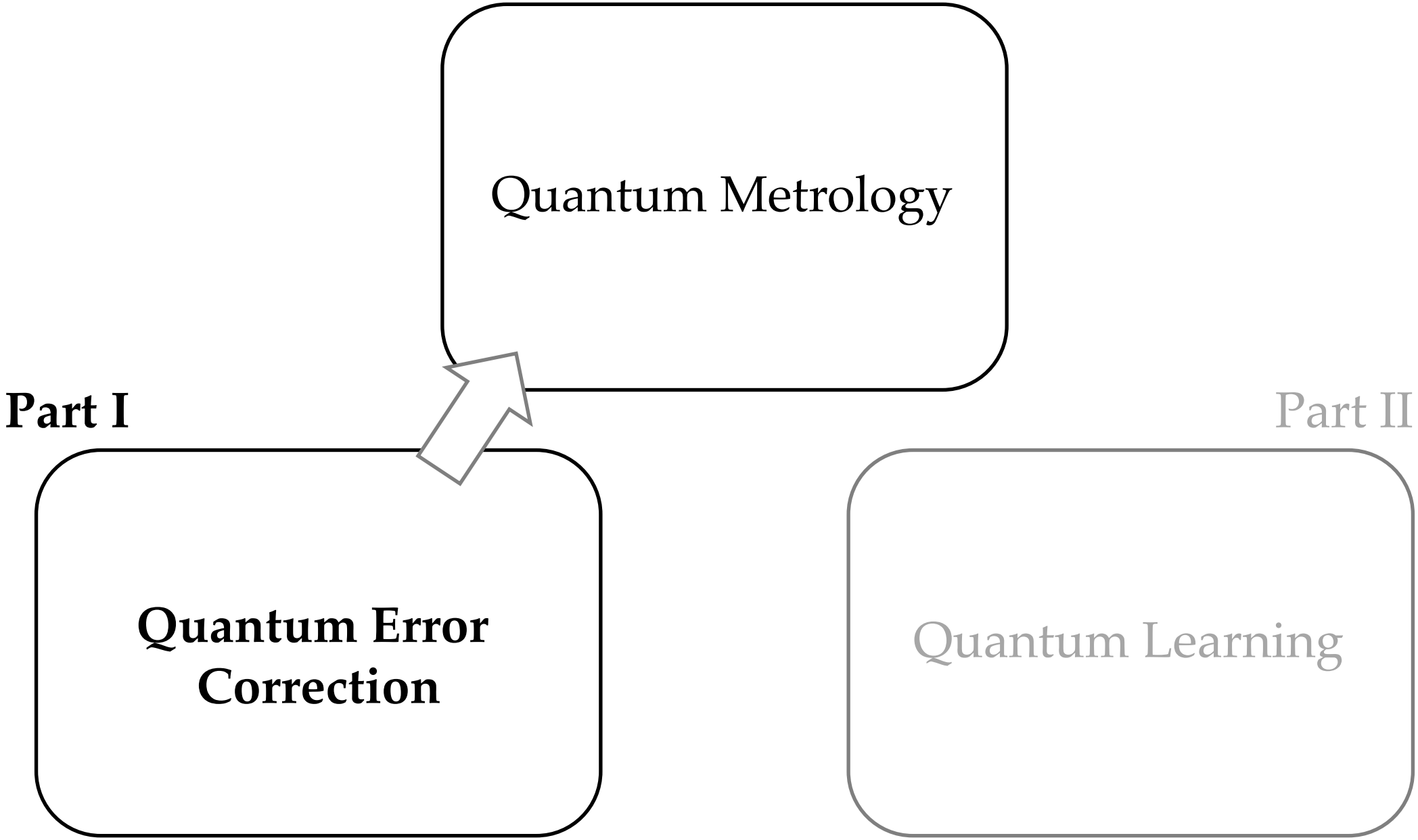
"Quantum" vs "Classical"



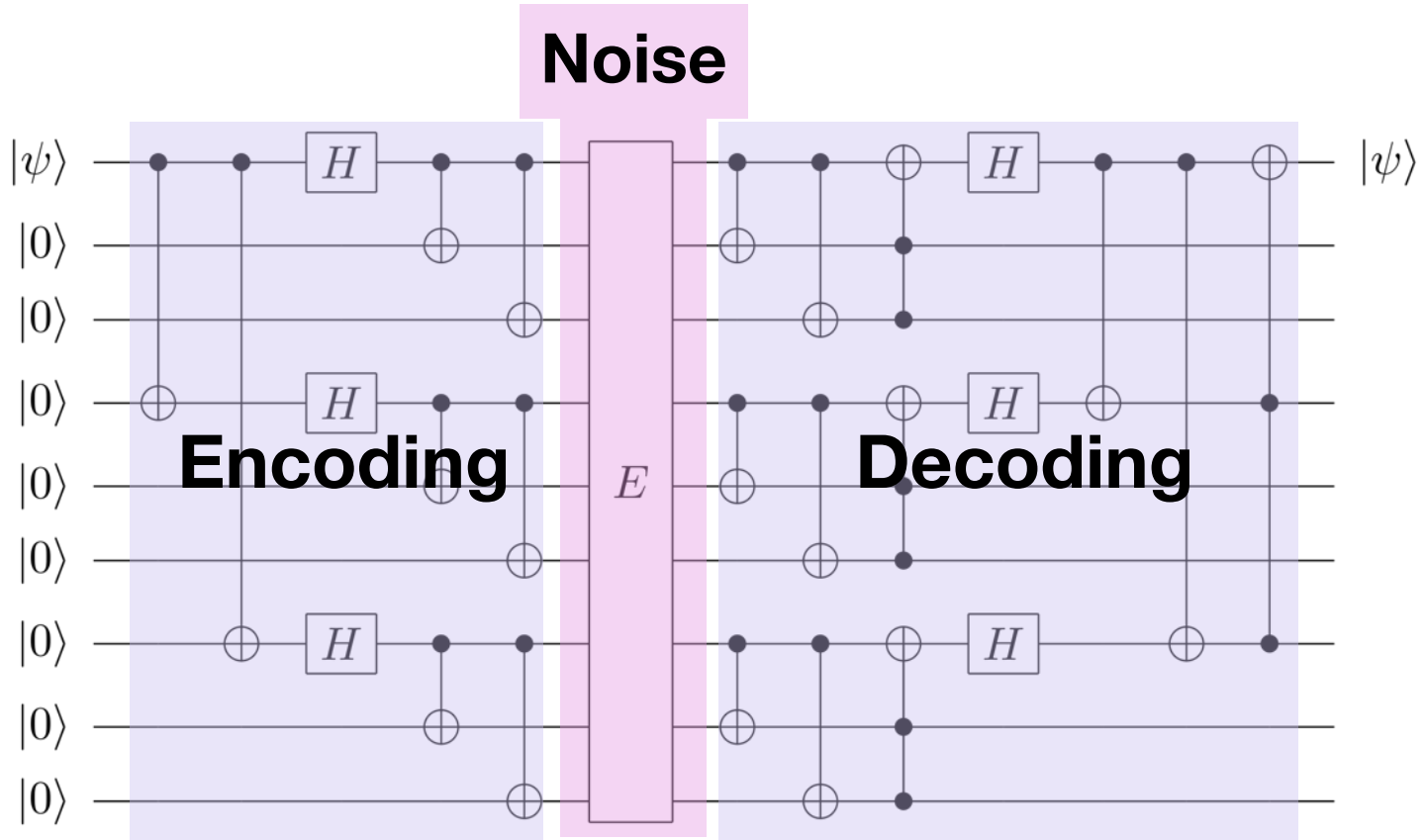
Oh, Chen, … , **SZ**, … , Andersen, Jiang, Preskill, PRL 2024

Liu, Brunel, … , **SZ**, … , Jiang, Preskill, Neergaard-Nielsen, Andersen, Science 2025


Quantum Metrology
Part I
Part II
Quantum Error Correction
Quantum Learning

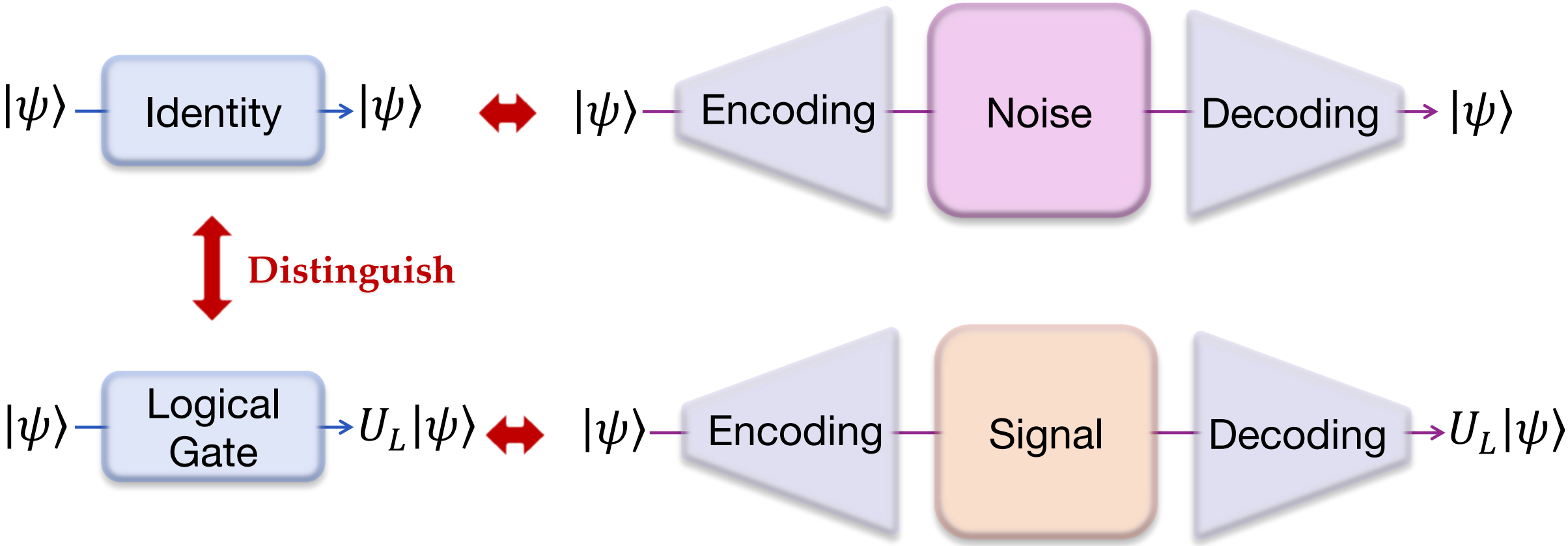

Quantum Metrology
Part I
Part II
Quantum Error Correction
Quantum Learning

Quantum error correction protects quantum information from noise, where logical qubits are encoded in multiple physical qubits.



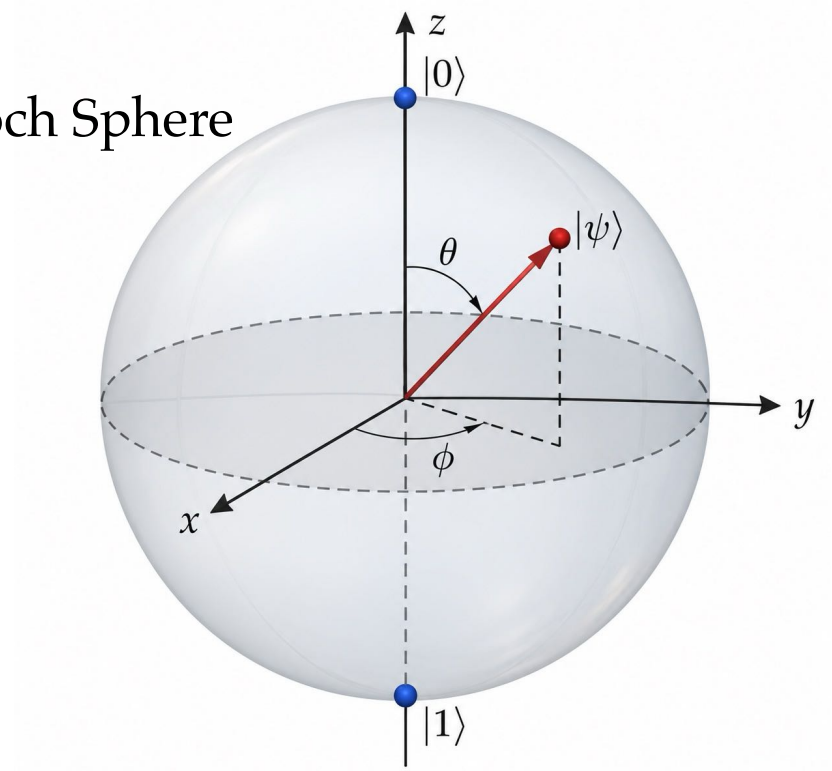
9-qubit Shor code

Correcting Noise + Preserving Signal



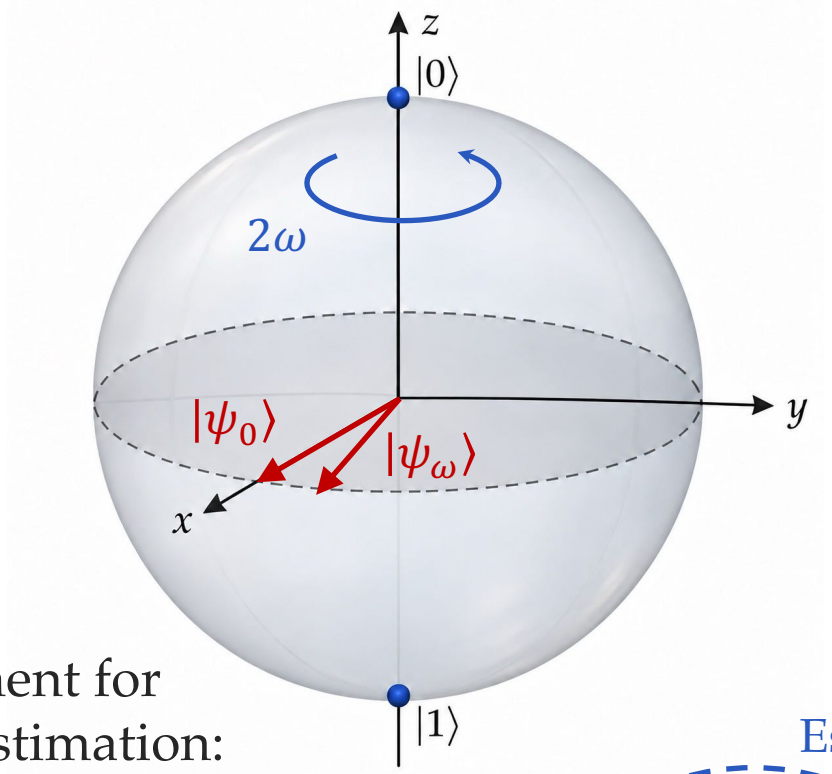
Phase Estimation

Bloch Sphere



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Pauli-Z Phase Rotation: $|\psi\rangle \rightarrow e^{-i\omega Z}|\psi\rangle$

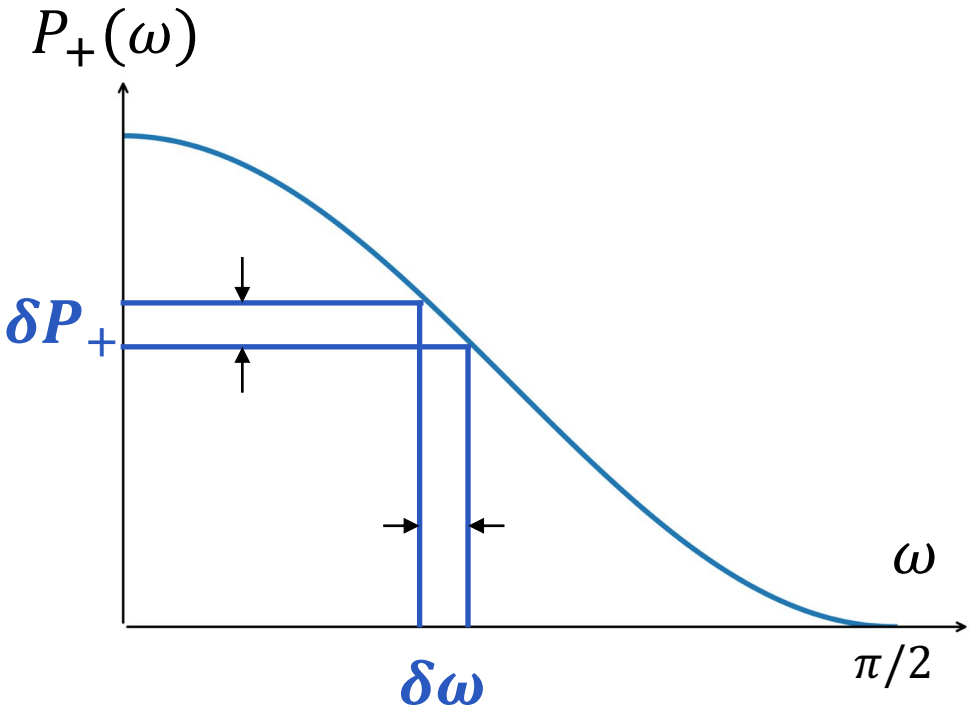


Ramsey Experiment for Phase Estimation:

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \rightarrow \frac{e^{-i\omega}|0\rangle + e^{i\omega}|1\rangle}{\sqrt{2}} \rightarrow \begin{cases} P_+ = \cos^2\omega \\ P_- = \sin^2\omega \end{cases}$$

Estimate ω

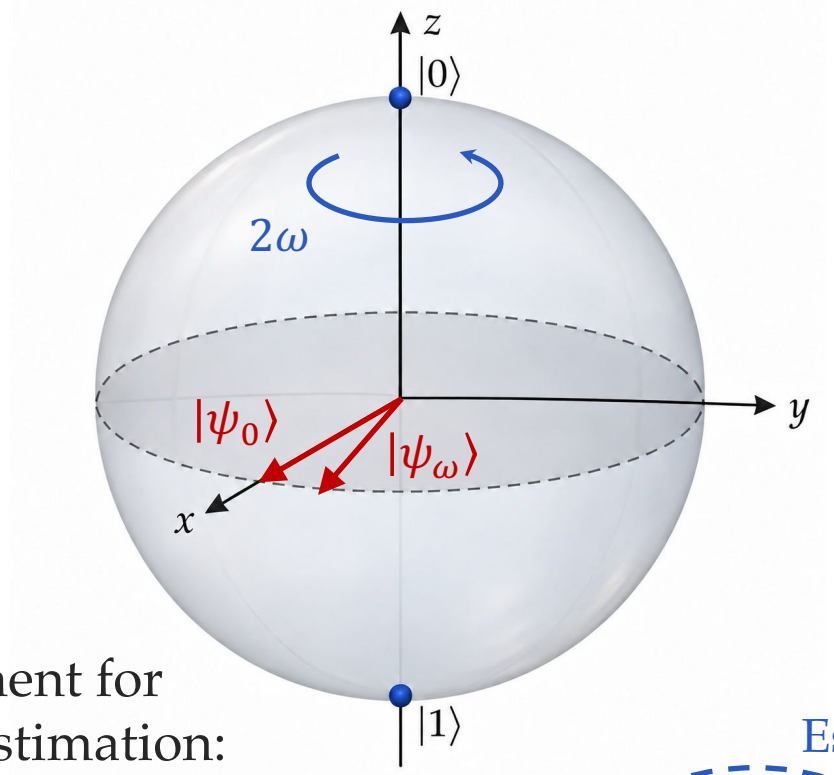
Phase Estimation





According to central limit theorem, repeating the experiment N times leads to **Standard quantum limit:** $\delta\omega \propto 1/\sqrt{N}$

Pauli-Z Phase Rotation: $|\psi\rangle \rightarrow e^{-i\omega Z}|\psi\rangle$



Ramsey Experiment for Phase Estimation:

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \rightarrow \frac{e^{-i\omega}|0\rangle + e^{i\omega}|1\rangle}{\sqrt{2}} \rightarrow \begin{cases} P_+ = \cos^2\omega \\ P_- = \sin^2\omega \end{cases}$$

Estimate ω

Phase Estimation

$$\mathcal{E}_\omega(\cdot) = e^{-i\omega Z}(\cdot)e^{i\omega Z}$$

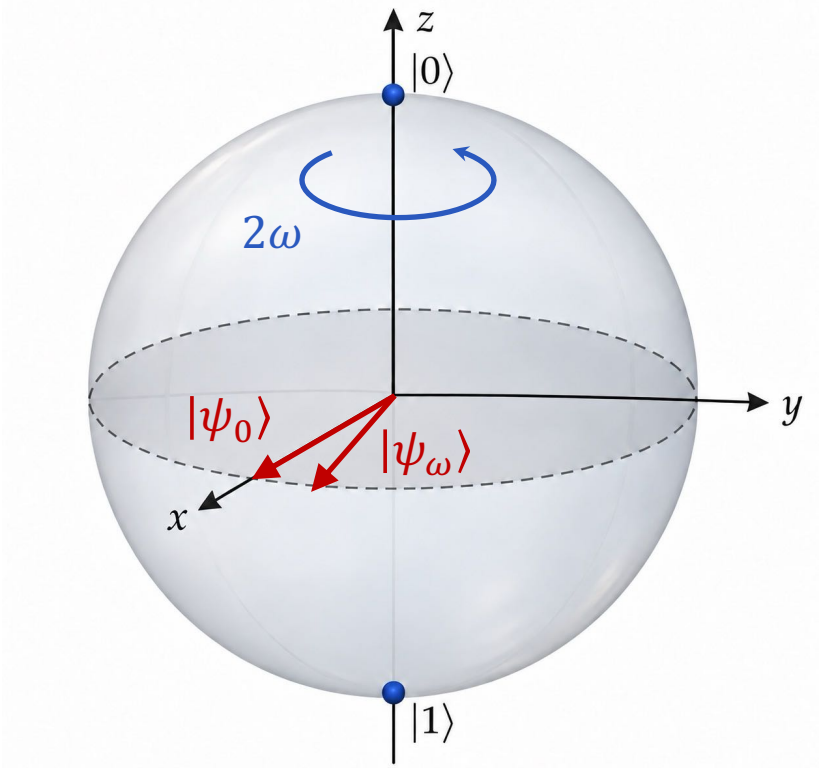


$$|\psi_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow |\psi_\omega\rangle = \frac{e^{-i\omega N}|0\rangle + e^{i\omega N}|1\rangle}{\sqrt{2}}$$

$$\begin{cases} P_+ = \cos^2 \omega N \\ P_- = \sin^2 \omega N \end{cases}$$

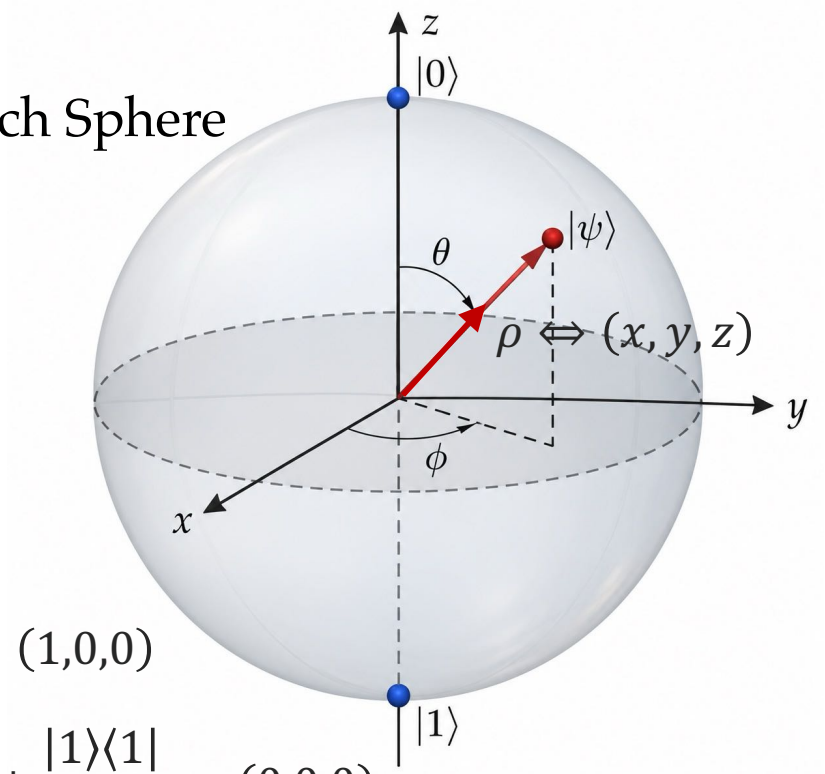
Heisenberg limit: $\delta\omega \propto 1/N$

Pauli-Z Phase Rotation: $|\psi\rangle \rightarrow e^{-i\omega Z}|\psi\rangle$



Bit-flip Noise

Bloch Sphere

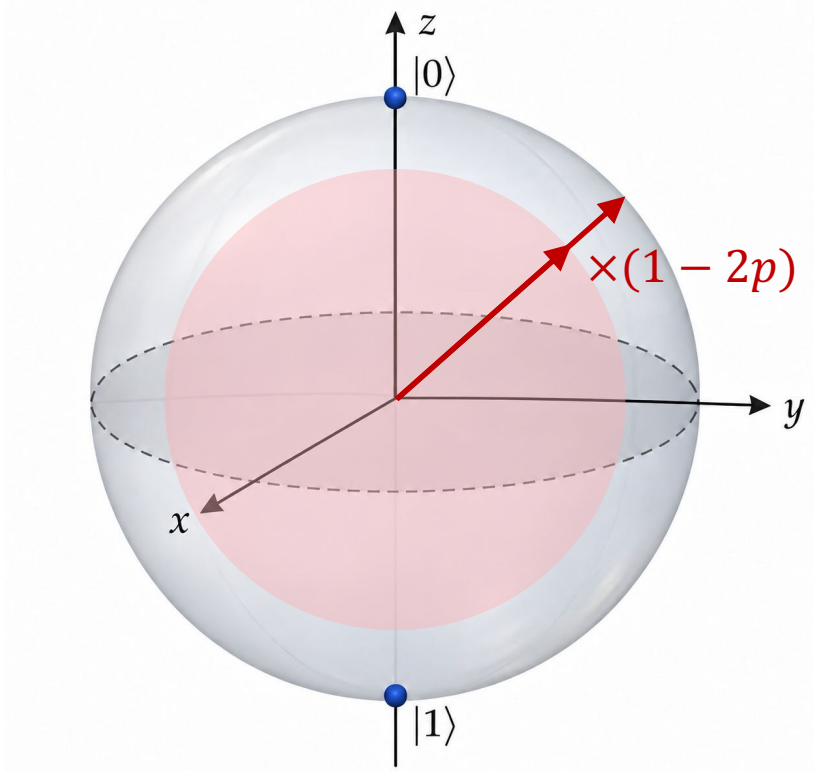


$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \Leftrightarrow (1,0,0)$$

$$\frac{\mathbb{I}}{2} = \frac{|0\rangle\langle 0|}{2} + \frac{|1\rangle\langle 1|}{2} \Leftrightarrow (0,0,0)$$

$$\rho = \frac{\mathbb{I} + x \cdot X + y \cdot Y + z \cdot Z}{2}$$

Pauli-X Bit-flip Noise: $\rho \to (1-p)\rho + pX\rho X$



$$(x, y, z) \to (x, (1-2p)y, (1-2p)z)$$

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$



[Ramsey Experiment]

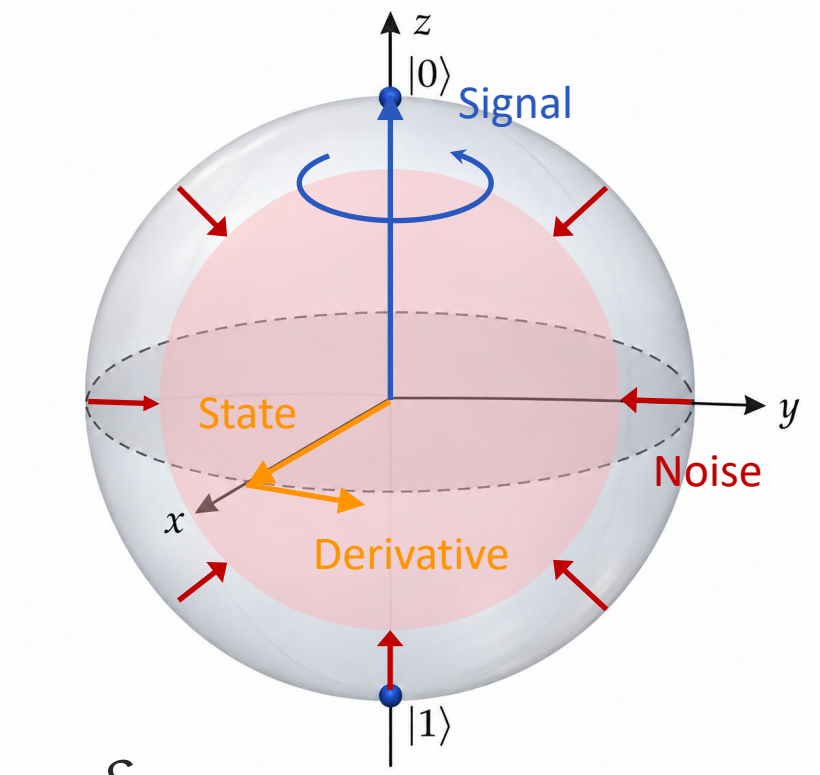
State unchanged:

$$(1,0,0) \xrightarrow{\mathcal{E}_0} (1,0,0) \quad \cdots\cdots \quad \xrightarrow{\mathcal{E}_0} (1,0,0)$$

Derivative decays:

$$(1,0,0) \xrightarrow{\partial_\omega \mathcal{E}_\omega|_{\omega=0}} (0,1-2p,0) \xrightarrow{\mathcal{E}_0} (0,(1-2p)^2,0) \quad \cdots\cdots \quad \xrightarrow{\mathcal{E}_0} (1,(1-2p)^N,0)$$

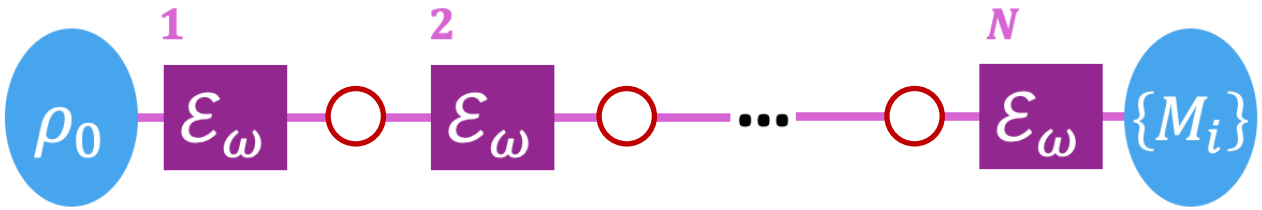
Phase Rotation + Bit-flip Noise



Estimation precision hits a floor as $N \to \infty$

Phase Estimation + Bit-flip Noise

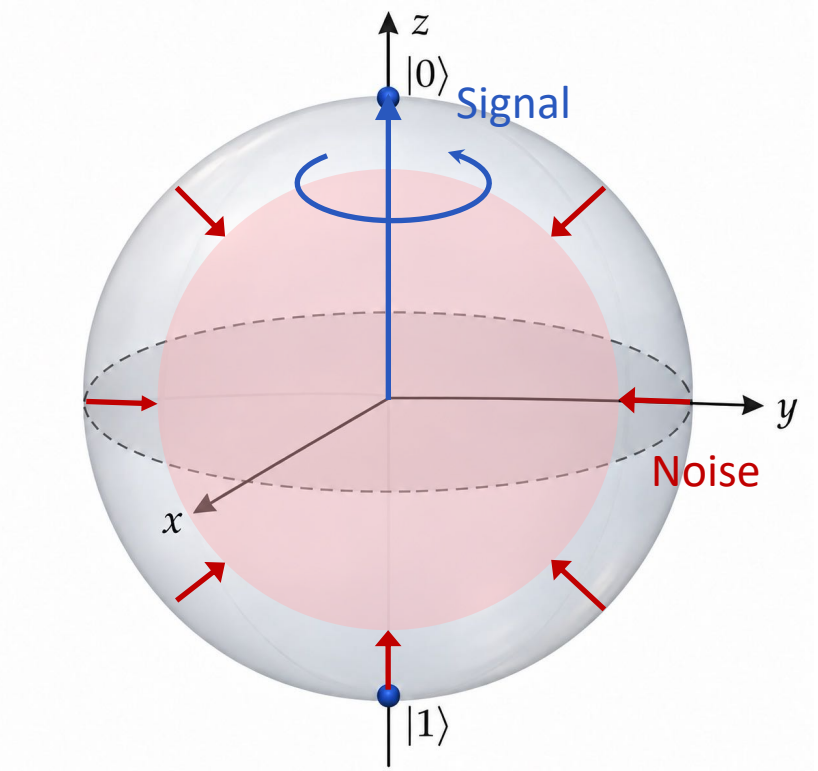
$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$



Standard quantum limit: $\delta\omega \propto 1/\sqrt{N}$

- Quadratically worse than **Heisenberg limit**
- Cannot be surpassed via arbitrary unitary controls
- Can be achieved via some unitary control

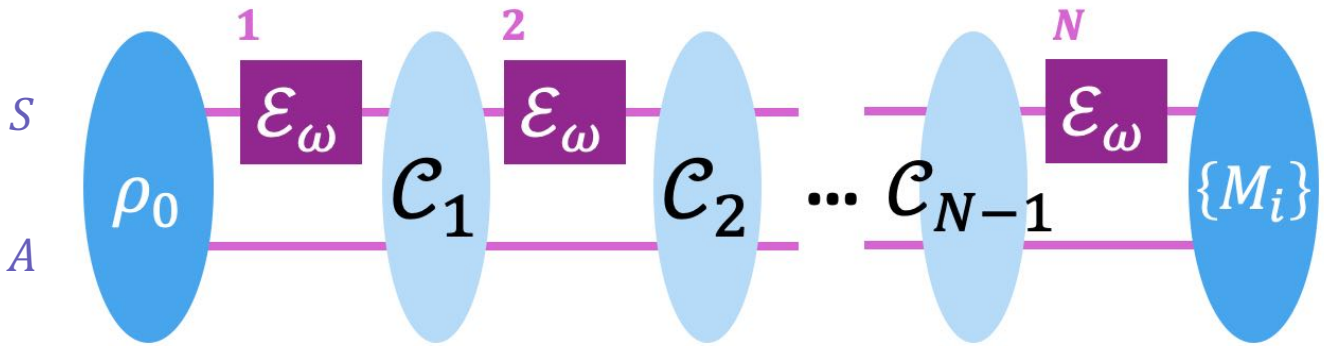
Phase Rotation + Bit-flip Noise



SZ, PRL 2024

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$



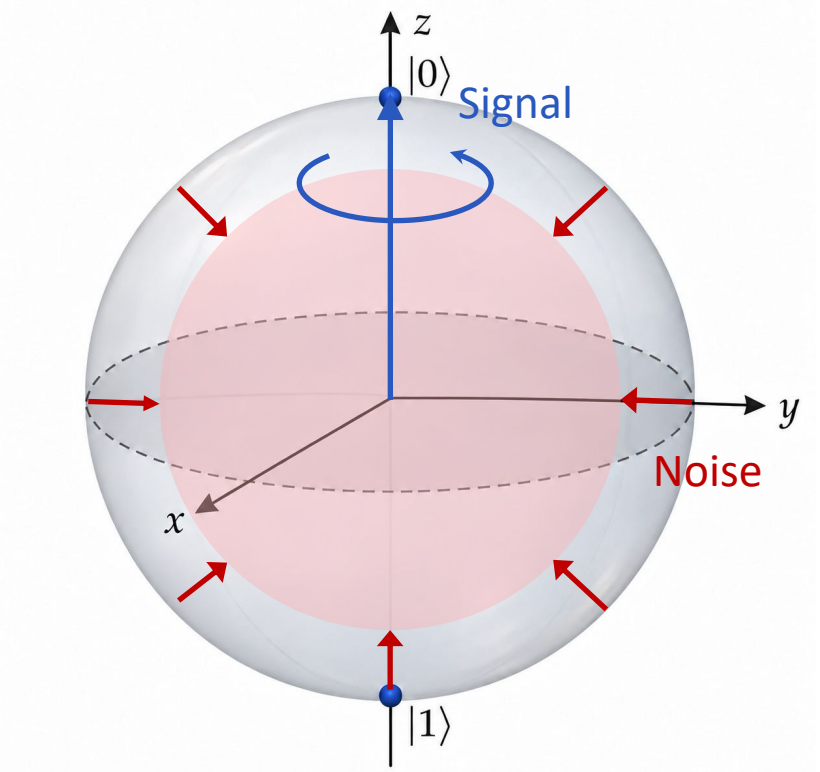
$$|\psi_0\rangle = \frac{|0\rangle_S + |1\rangle_S}{\sqrt{2}}$$

Repetition Code

Encoding

$$|\psi_0\rangle = \frac{|0_L\rangle + |1_L\rangle}{\sqrt{2}} = \frac{|0\rangle_S|0\rangle_A + |1\rangle_S|1\rangle_A}{\sqrt{2}}$$

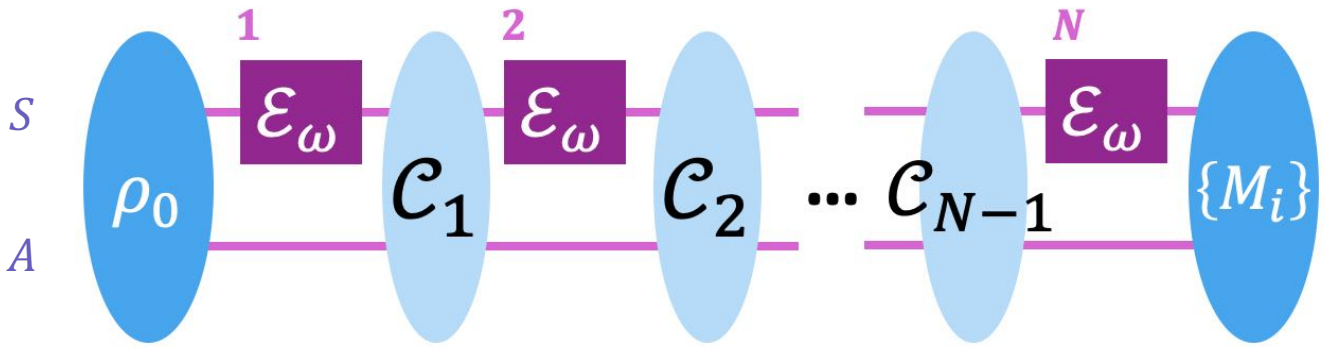
Phase Rotation + Bit-flip Noise



Dur et al. PRL 2014; Kessler et al. PRL 2014; Arrad et al. PRL 2014

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$

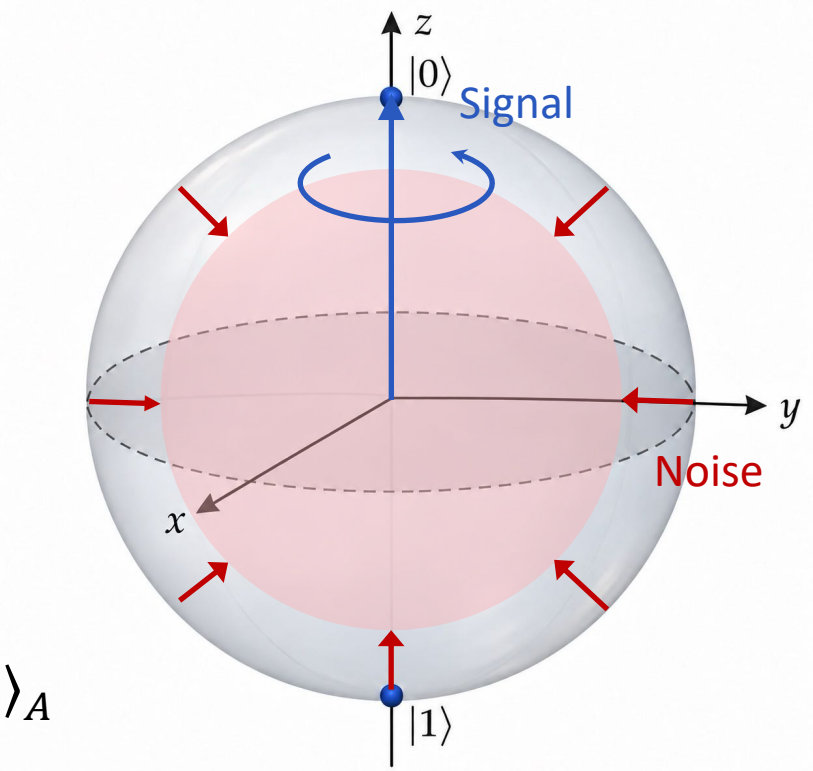


$$|0\rangle_S|0\rangle_A \xrightarrow{X_S} |1\rangle_S|0\rangle_A \xrightarrow{Z_SZ_A} z_sz_a = -1 \xrightarrow{X_S} |0\rangle_S|0\rangle_A$$

$$|1\rangle_S|1\rangle_A \xrightarrow{X_S} |0\rangle_S|1\rangle_A \xrightarrow{Z_SZ_A} z_sz_a = -1 \xrightarrow{X_S} |1\rangle_S|1\rangle_A$$

Error Detection Correction

Phase Rotation + Bit-flip Noise

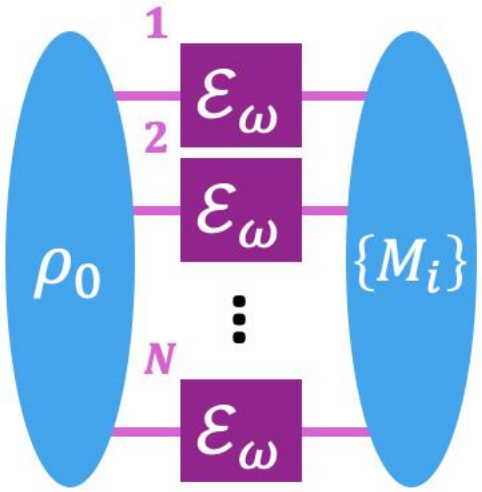


Heisenberg limit recovered

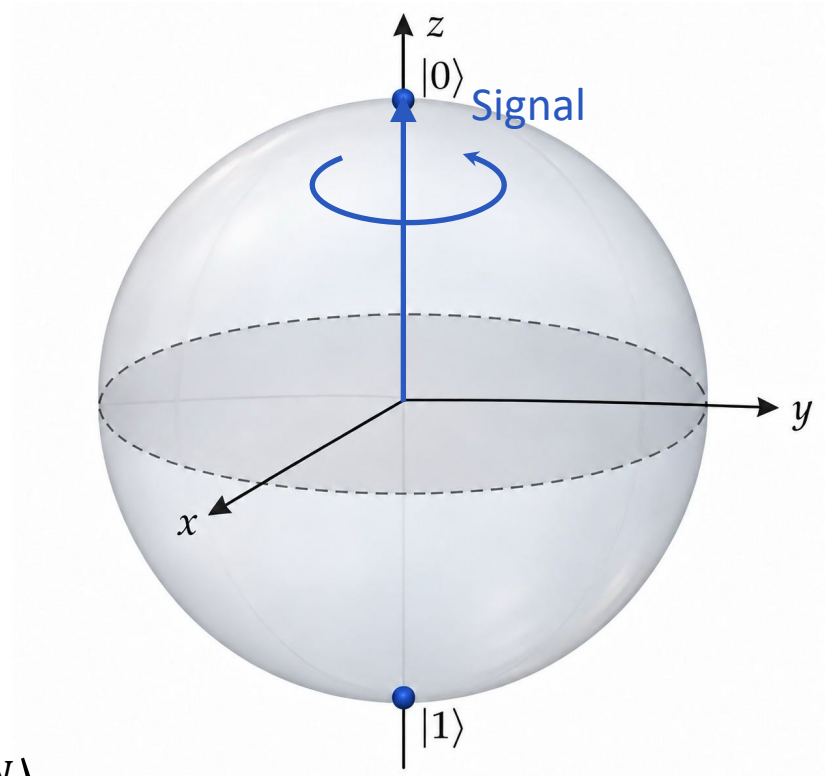
Dur et al. PRL 2014; Kessler et al. PRL 2014; Arrad et al. PRL 2014

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = e^{-i\omega Z}(\cdot)e^{i\omega Z}$$



Phase Rotation



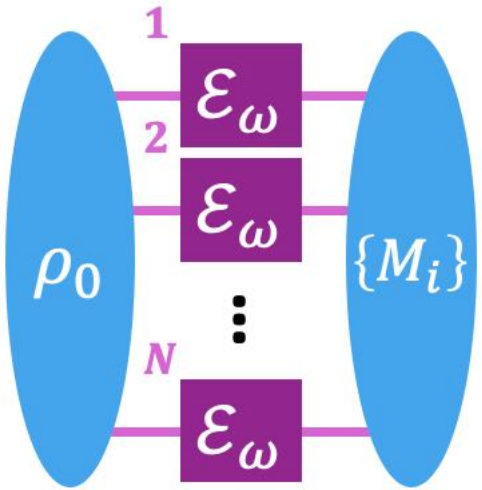
$$|\psi_0\rangle = \frac{|0^{\otimes N}\rangle + |1^{\otimes N}\rangle}{\sqrt{2}} \rightarrow |\psi_\omega\rangle = \frac{e^{-i\omega N}|0^{\otimes N}\rangle + e^{i\omega N}|1^{\otimes N}\rangle}{\sqrt{2}}$$

Heisenberg limit: $\delta\omega \propto 1/N$

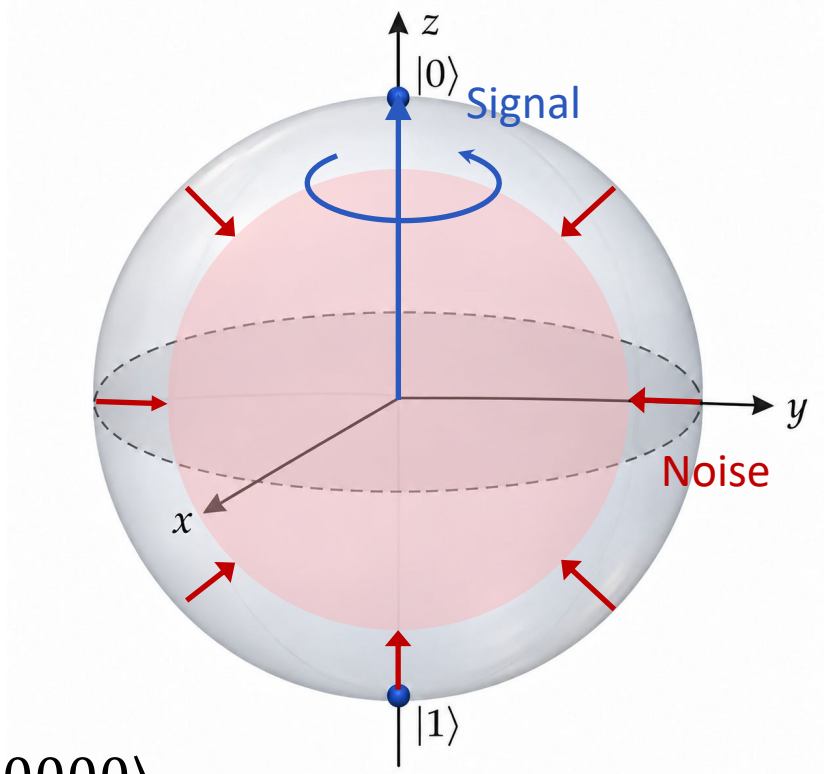
Dur et al. PRL 2014; Kessler et al. PRL 2014; Arrad et al. PRL 2014

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$



Phase Rotation + Bit-flip Noise



12, 23, 34, 45, 51

$$|00000\rangle \xrightarrow{X_2X_3} |01100\rangle \xrightarrow{Z_iZ_{i+1}} (-,+,-,+,+) \xrightarrow{X_2X_3} |00000\rangle$$

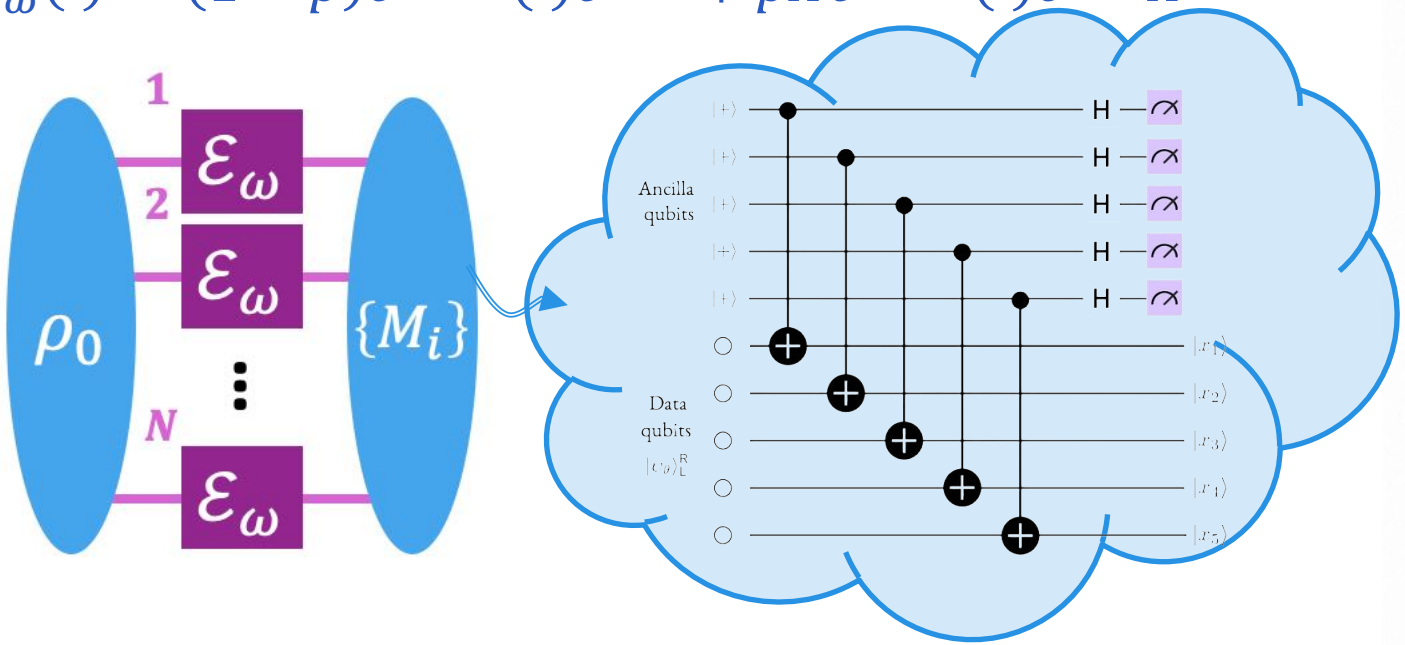
$$|11111\rangle \xrightarrow{X_2X_3} |10011\rangle \xrightarrow{Z_iZ_{i+1}} (-,+,-,+,+) \xrightarrow{X_2X_3} |11111\rangle$$

Error Detection Correction

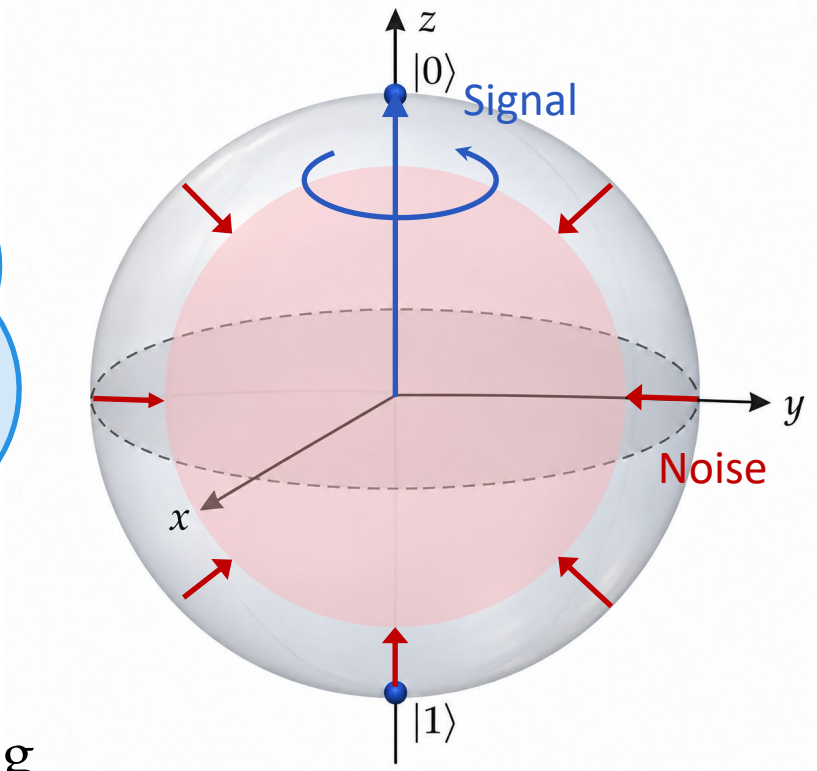
Dur et al. PRL 2014; Kessler et al. PRL 2014; Arrad et al. PRL 2014

Phase Estimation + Bit-flip Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pXe^{-i\omega Z}(\cdot)e^{i\omega Z}X$$



Phase Rotation + Bit-flip Noise



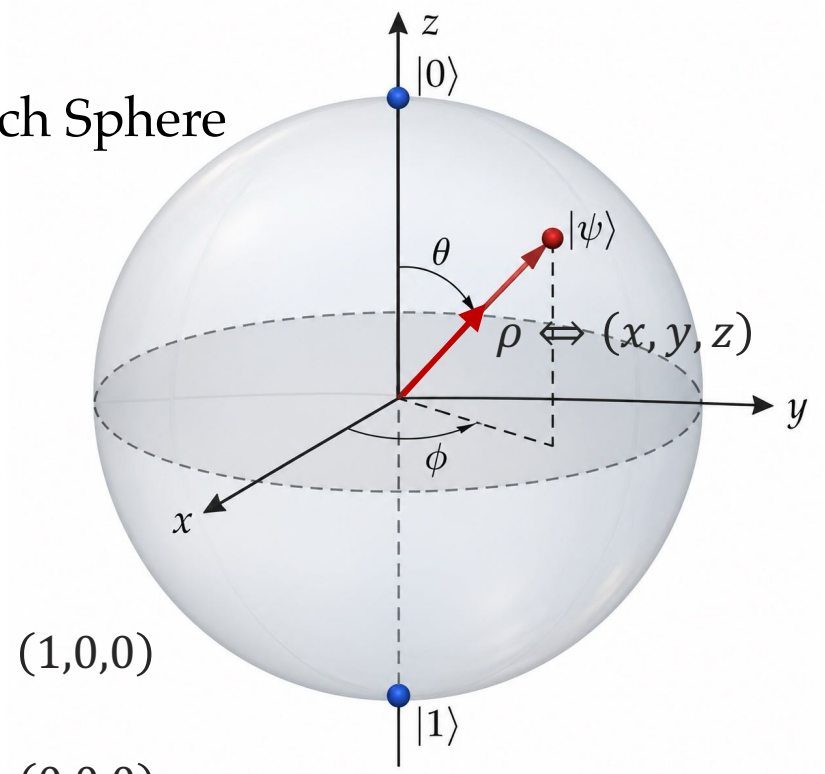
Heisenberg limit is recoverable even when considering

- State preparation and measurement noise
- Bit-flip noise after every gate (or idle) operations

Sahu, Xu, **SZ**, 2026

Dephasing Noise

Bloch Sphere

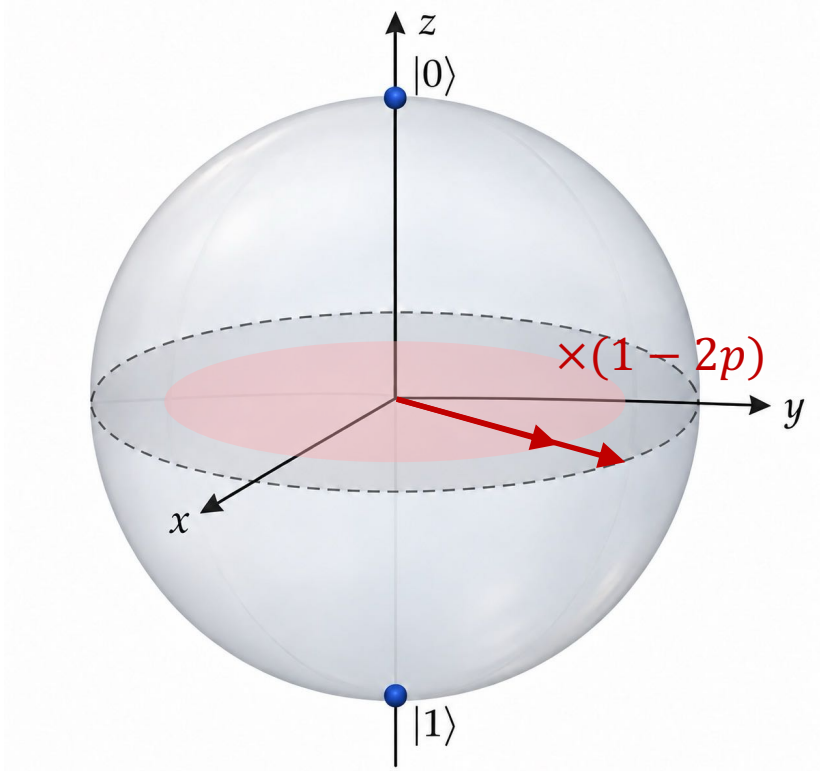


$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \Leftrightarrow (1,0,0)$$

$$\frac{\mathbb{I}}{2} \Leftrightarrow (0,0,0)$$

$$\rho = \frac{\mathbb{I} + x \cdot X + y \cdot Y + z \cdot Z}{2}$$

Pauli-Z Dephasing Noise: $\rho \to (1-p)\rho + pZ\rho Z$



$$(x, y, z) \to ((1-2p)x, (1-2p)y, z)$$

Phase Estimation + Dephasing Noise

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pZe^{-i\omega Z}(\cdot)e^{i\omega Z}Z$$



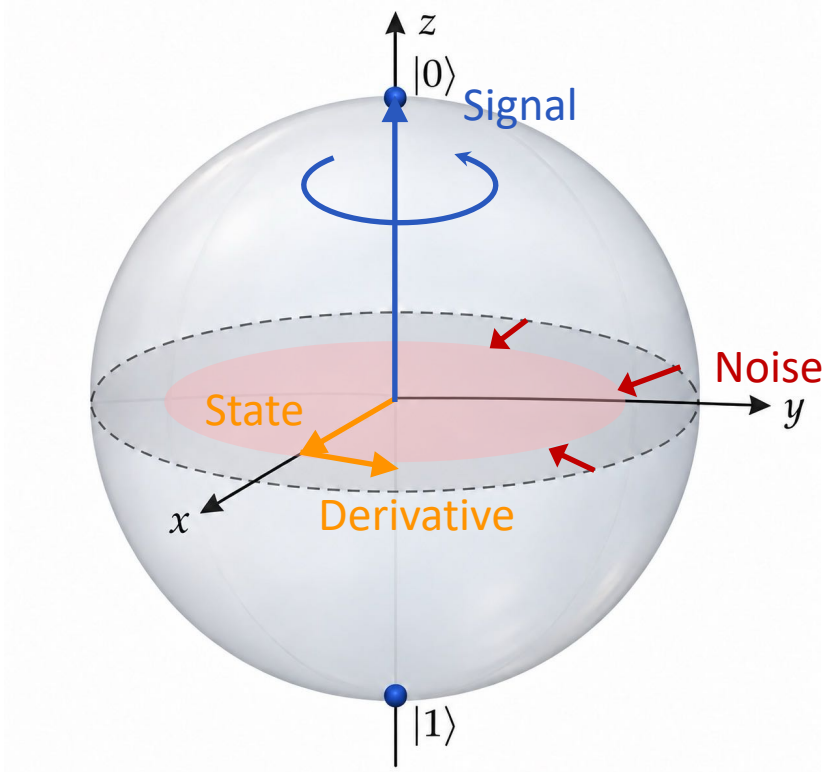
[Ramsey Experiment]

$$|\psi_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = (1,0,0)$$

$$\rightarrow \mathbf{(1-2p)^N}(\cos 2N\omega\,, \sin 2N\omega\,, 0)$$

State becomes maximally mixed as $N \rightarrow \infty$

Pauli-Z Dephasing Noise: $\rho \rightarrow (1-p)\rho + pZ\rho Z$

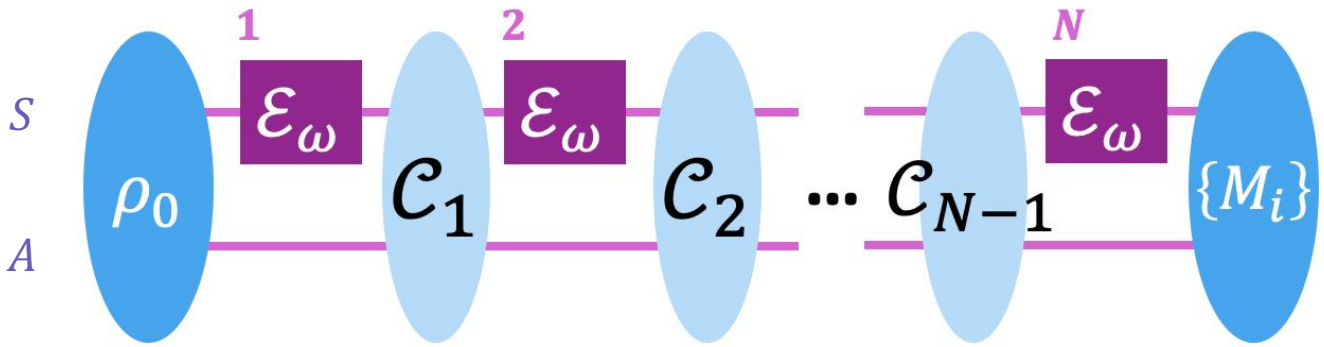


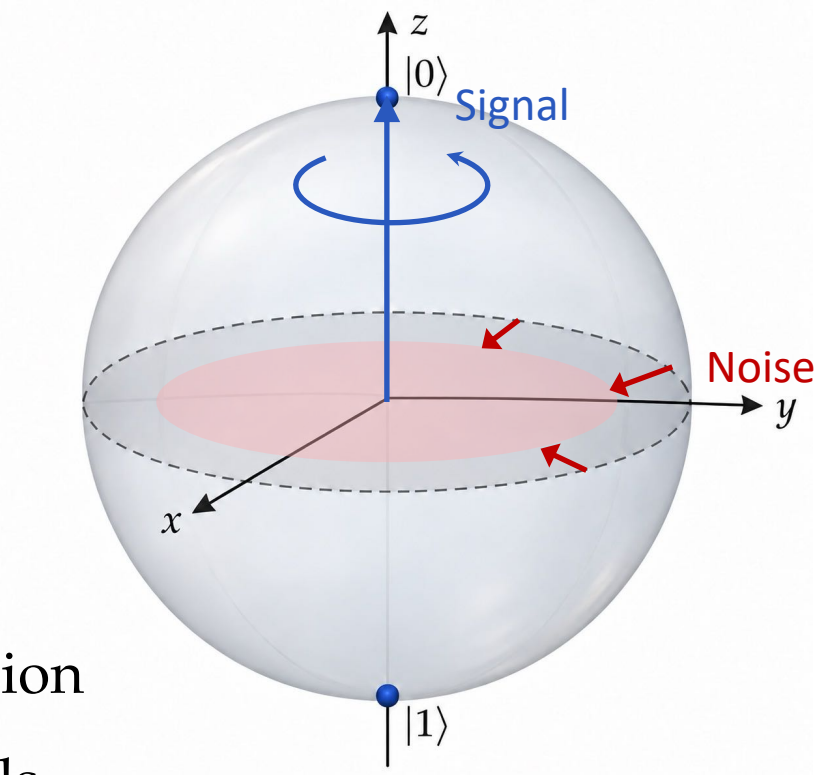
$$(x, y, z) \rightarrow ((1-2p)x, (1-2p)y, z)$$

Phase Estimation + Dephasing Noise

Pauli-Z Dephasing Noise: $\rho \to (1-p)\rho + pZ\rho Z$

$$\mathcal{E}_\omega(\cdot) = (1-p)e^{-i\omega Z}(\cdot)e^{i\omega Z} + pZe^{-i\omega Z}(\cdot)e^{i\omega Z}Z$$





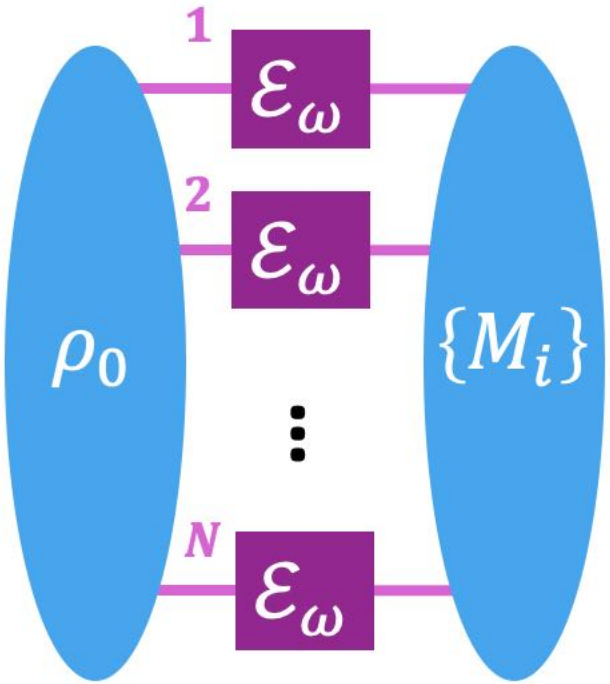
- **Standard quantum limit** is the best possible precision
- Cannot be surpassed via arbitrary quantum controls, including Quantum Error Correction

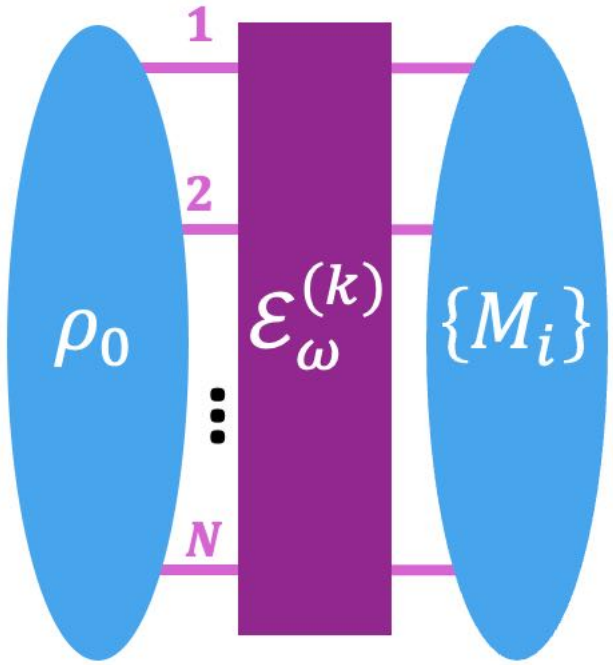
$$(x, y, z) \to ((1-2p)x, (1-2p)y, z)$$

- Signal is **indistinguishable** from noise

Demkowicz-Dobrzański et al., NCOMMS 2012 & PRX 2017, Escher et al., NPhys 2011, , **SZ** et al., NCOMMS 2018

Many-body Hamiltonian Estimation





Single-qubit Hamiltonian

$H = \omega \sum_{i=1}^{N} Z_i,\ \delta\omega \propto 1/N$

Heisenberg scaling

***k*-body Hamiltonian**

$H = \omega \sum_{(i_1,i_2,\ldots i_k)} Z_{i_1} Z_{i_2} \cdots Z_{i_k},\ \delta\omega \propto 1/N^k$

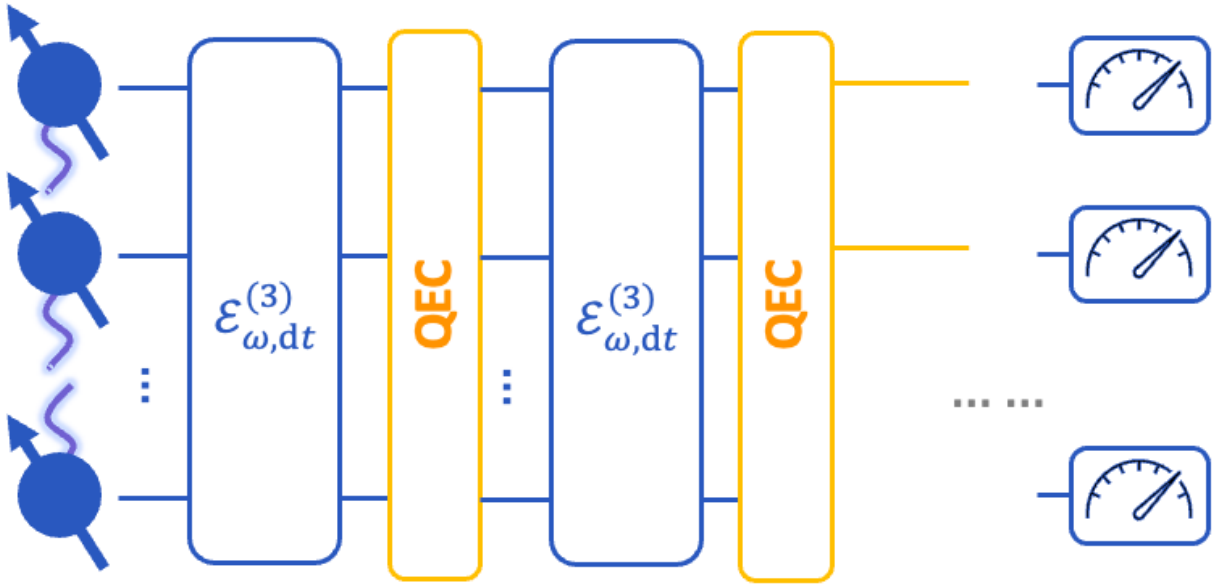
Super-Heisenberg scaling

Boixo et al. PRL 2007

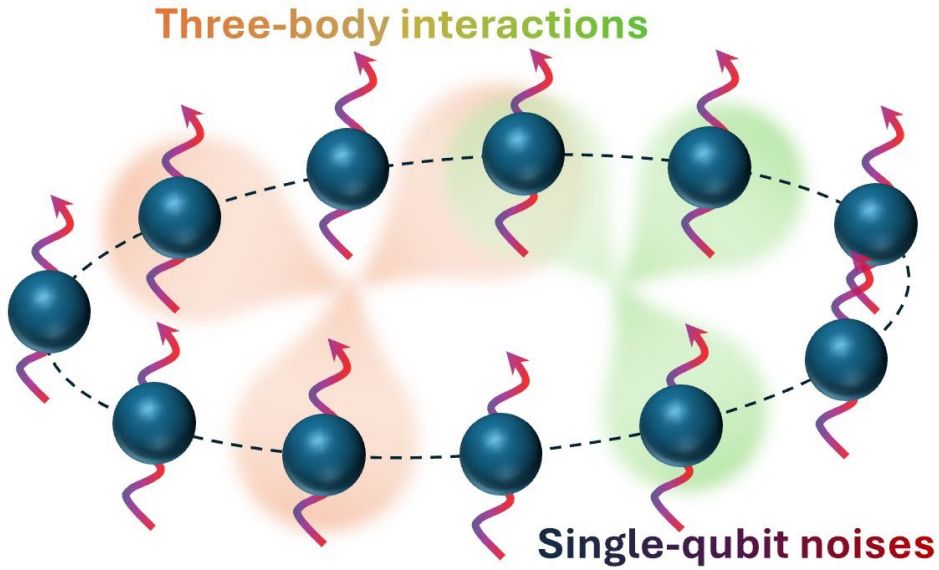
Many-body Hamiltonian Estimation + Single-qubit Noise

$k = 3$. Use $[[N, 1, 3]]$ QEC code:

- **Signal** ⇒ Logical operators of form $Z_{i_1} Z_{i_2} Z_{i_3}$
- **Noise** ⇒ Can correct one-qubit error



Heisenberg limit: $\delta\omega \propto 1/t$



3-body Hamiltonian

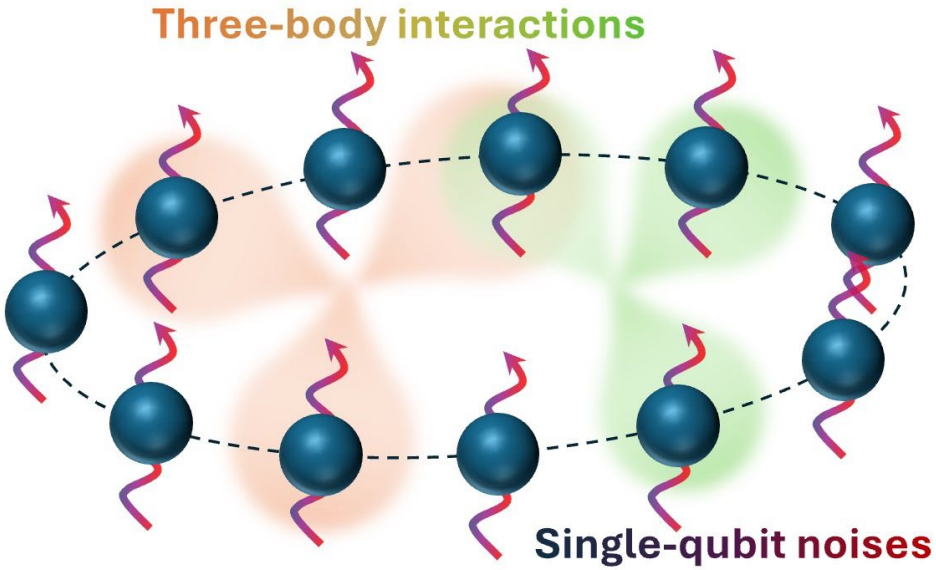
$$H = \omega \sum_{(i_1,i_2,i_3)} Z_{i_1} Z_{i_2} Z_{i_3}, \quad \delta\omega \propto 1/N^3$$

Antu, SZ, Quantum 2025

Many-body Hamiltonian Estimation + Single-qubit Noise

$k = 3$. Use $[[N, 1, 3]]$ QEC code:

- **Signal** ⇒ Logical operators of form $Z_{i_1} Z_{i_2} Z_{i_3}$
- **Noise** ⇒ Can correct one-qubit error



- Thin Surface Code: $\delta\omega \propto 1/N$
- Quantum Reed-Muller Code: $\delta\omega \propto 1/N^2$
- Generalized Shor Code: $\delta\omega \propto 1/N^3$

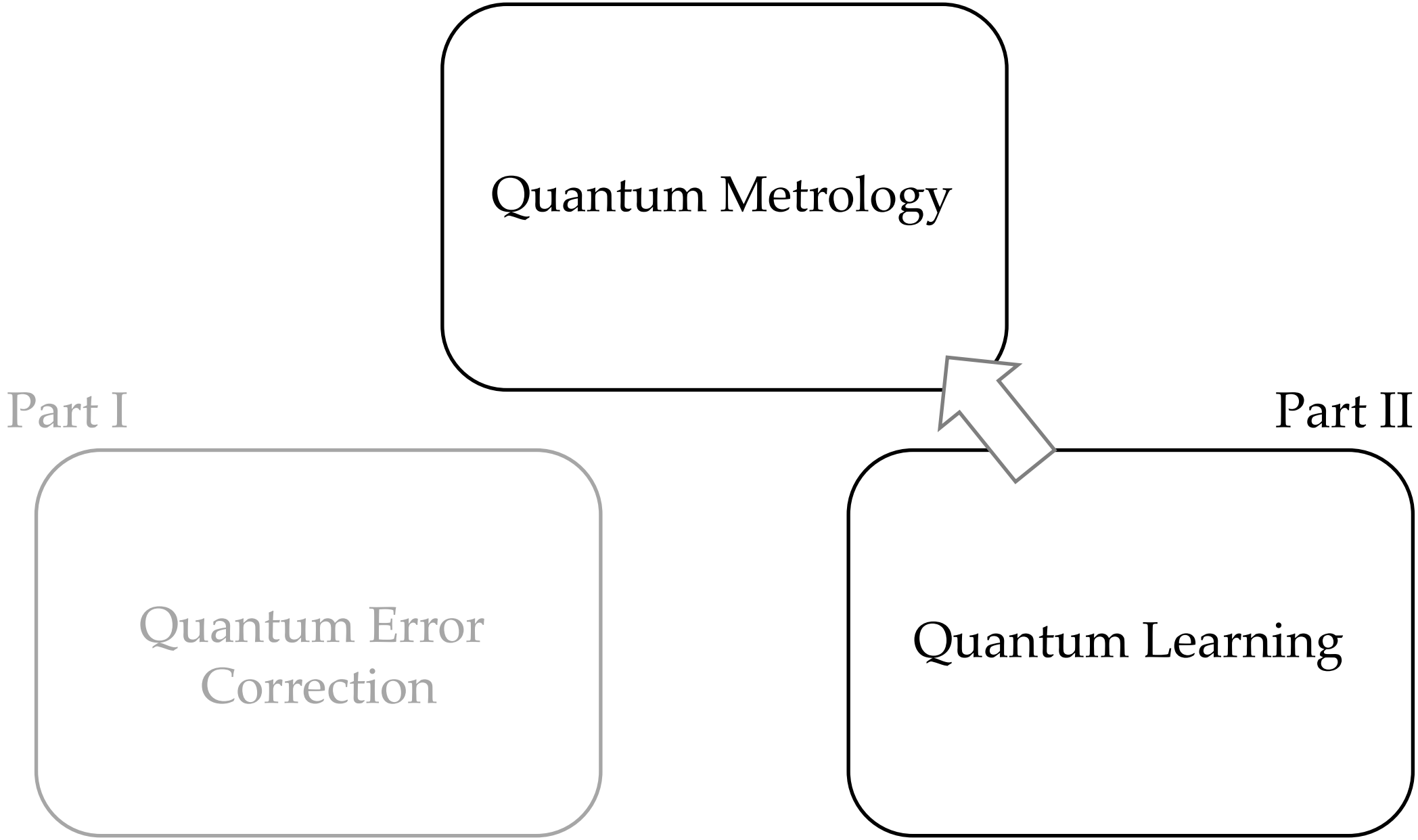
3-body Hamiltonian

$$H = \omega \sum_{(i_1,i_2,i_3)} Z_{i_1} Z_{i_2} Z_{i_3}, \quad \delta\omega \propto 1/N^3$$

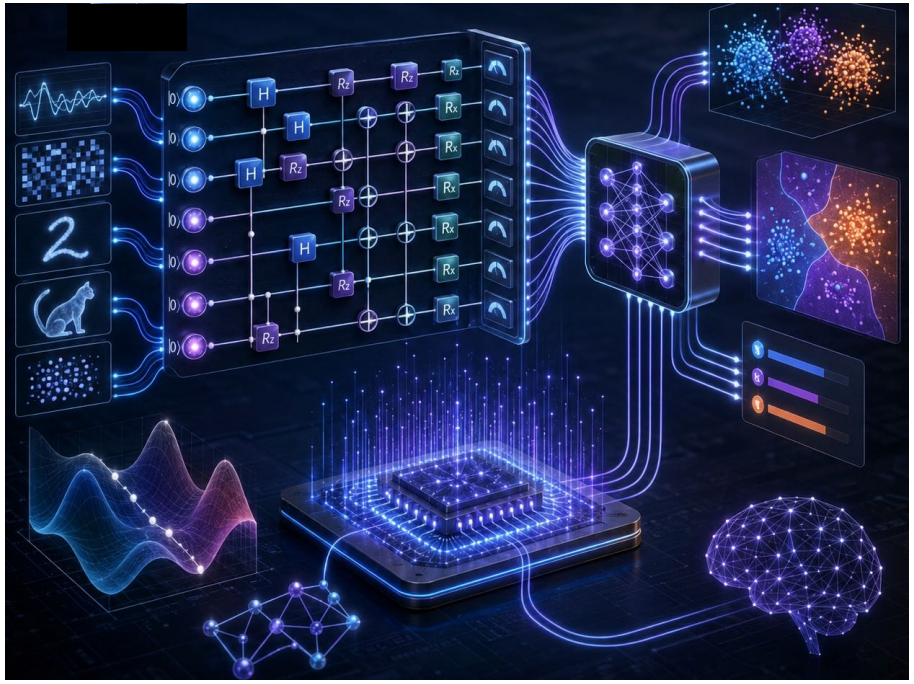
Antu, SZ, Quantum 2025

Summary of Part I

- Without quantum noise, **Heisenberg limit** usually represents the ultimate estimation limit in quantum metrology.
- With quantum noise, the estimation limit degrades to **Standard quantum limit**.
- **Quantum error correction** can recover the Heisenberg limit when signal and noise are distinguishable.
- **No quantum controls** can surpass the standard quantum limit when signal and noise are indistinguishable.
- For interacting Hamiltonians, quantum error correction can suppress low-weight noise while preserving signals encoded in higher-weight interactions.


Quantum Metrology
Part I
Part II
Quantum Error Correction
Quantum Learning

Quantum Learning



Input: $\rho^{\otimes N}$

Output: $f_1(\rho), f_2(\rho), \ldots, f_m(\rho)$

- State Tomography:

$$\hat{\rho} \approx \rho$$

- Observable learning (Shadow Tomography):

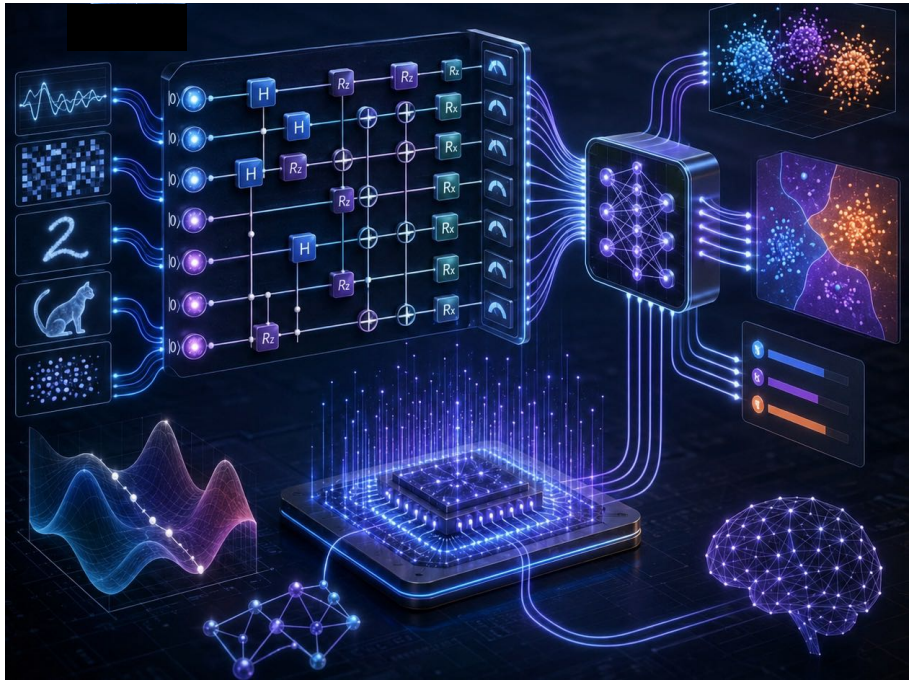
$$\mathrm{Tr}(\rho O_1), \ldots, \mathrm{Tr}(\rho O_m)$$

- State Certification:

$$\begin{cases} \text{Yes}, & \rho \approx \sigma \\ \text{No}, & \rho \not\approx \sigma \end{cases}$$

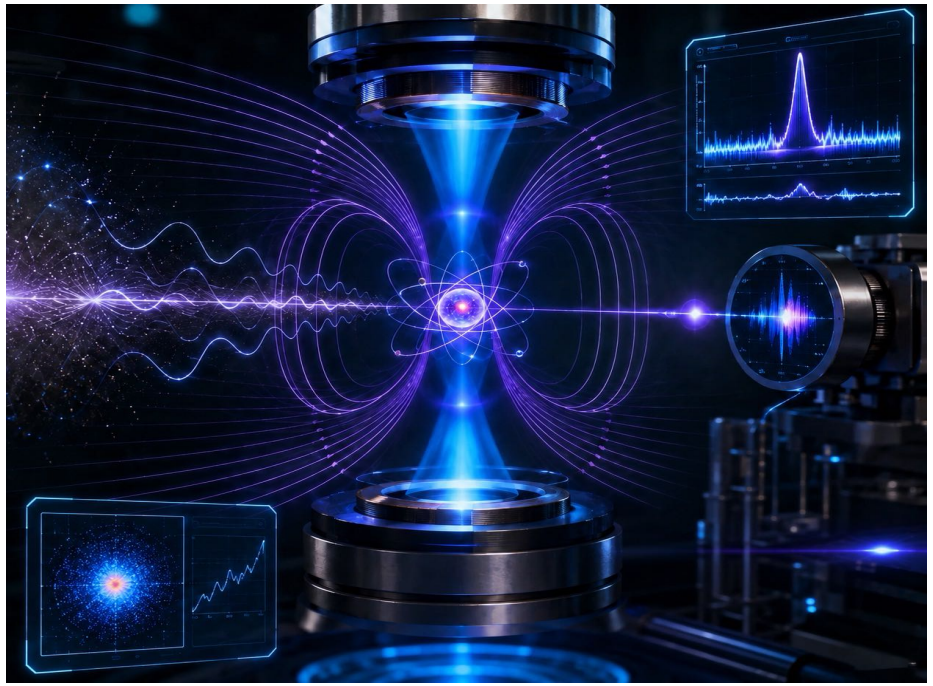
……

Quantum Learning vs. Multi-parameter Metrology



Input: $\rho^{\otimes N}$

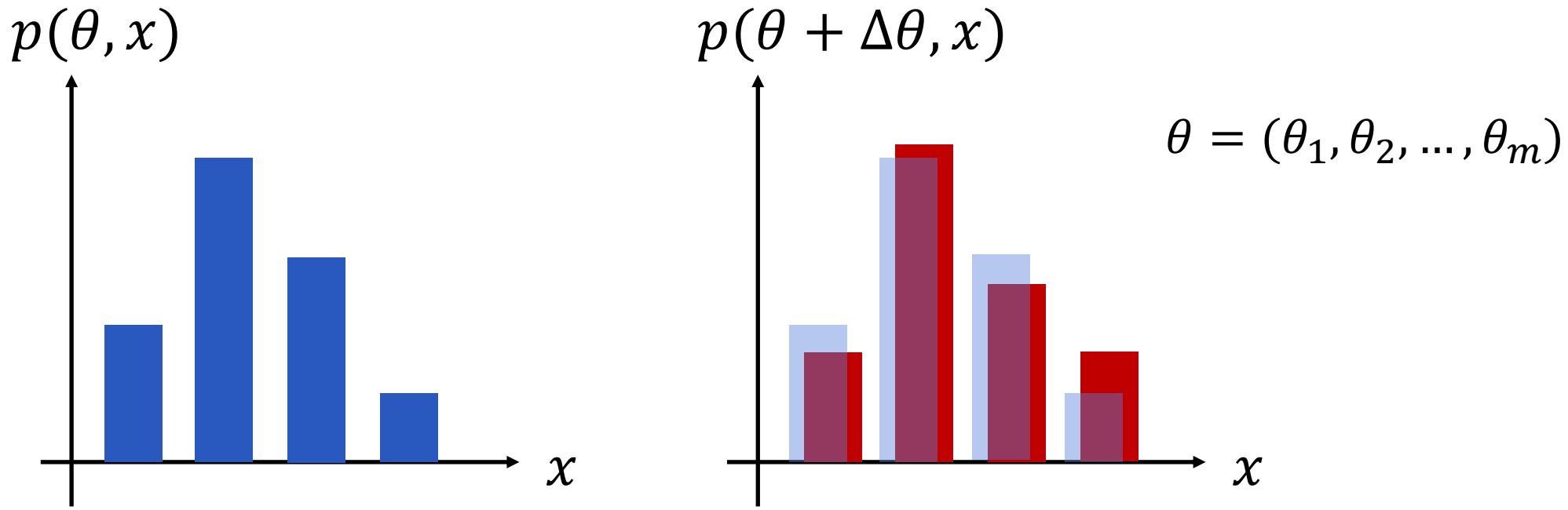
Output: $f_1(\rho), f_2(\rho), \ldots, f_m(\rho)$



Input: $\rho_\theta^{\otimes N}$

Output: $\theta_1, \ldots, \theta_m$

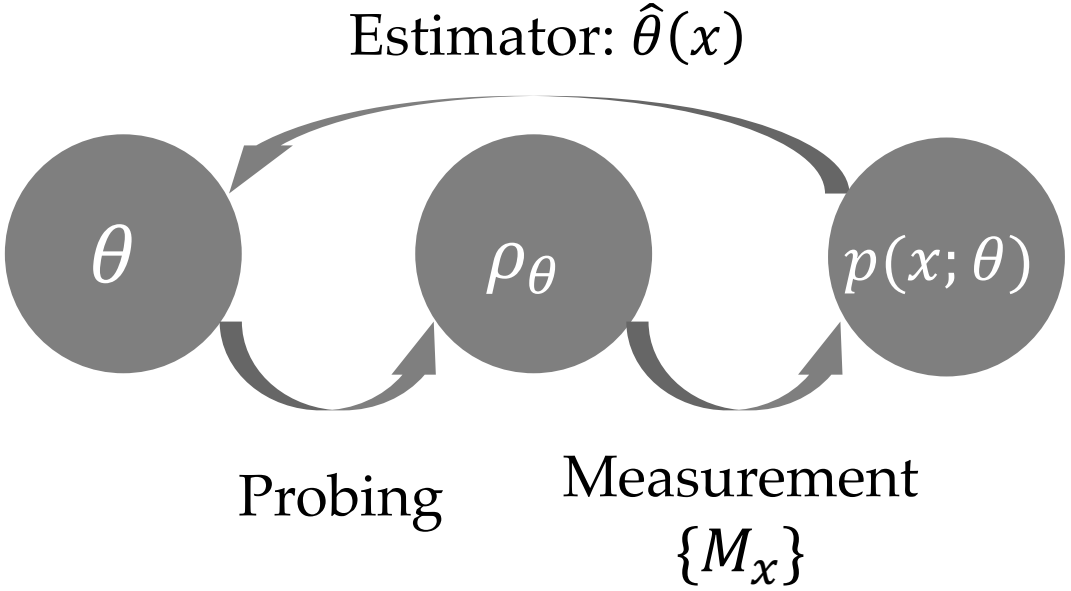
Multi-parameter Estimation



Mean squared error matrix (MSEM):

$$V_{ij}(\theta) = \mathbb{E}_x[(\widehat{\theta}_i(x) - \theta_i)(\widehat{\theta}_j(x) - \theta_j)]$$

(Quantum) Cramér–Rao bound



- Quantum Cramér–Rao bound:

$$\boldsymbol{V(\theta) \geq J^{-1}(\rho_\theta)},$$

J: Quantum Fisher information matrix

- (Classical) Cramér–Rao bound:

$$\boldsymbol{V(\theta) \geq I^{-1}(\rho_\theta, M)},$$

I: Fisher information matrix

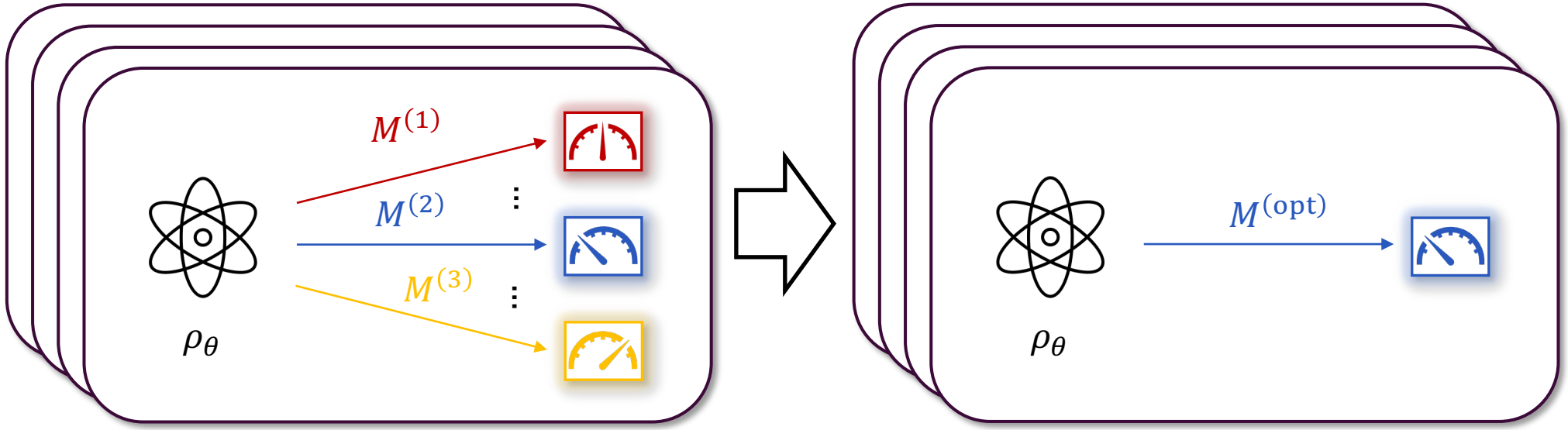
- Attainability:

Optimal Measurement?

Optimal Estimator $\approx$ $\not\approx$

$$\boldsymbol{V(\theta) \geq I^{-1}(\rho_\theta, M) \geq J^{-1}(\rho_\theta)}$$

Single-parameter Estimation



Fisher Information:

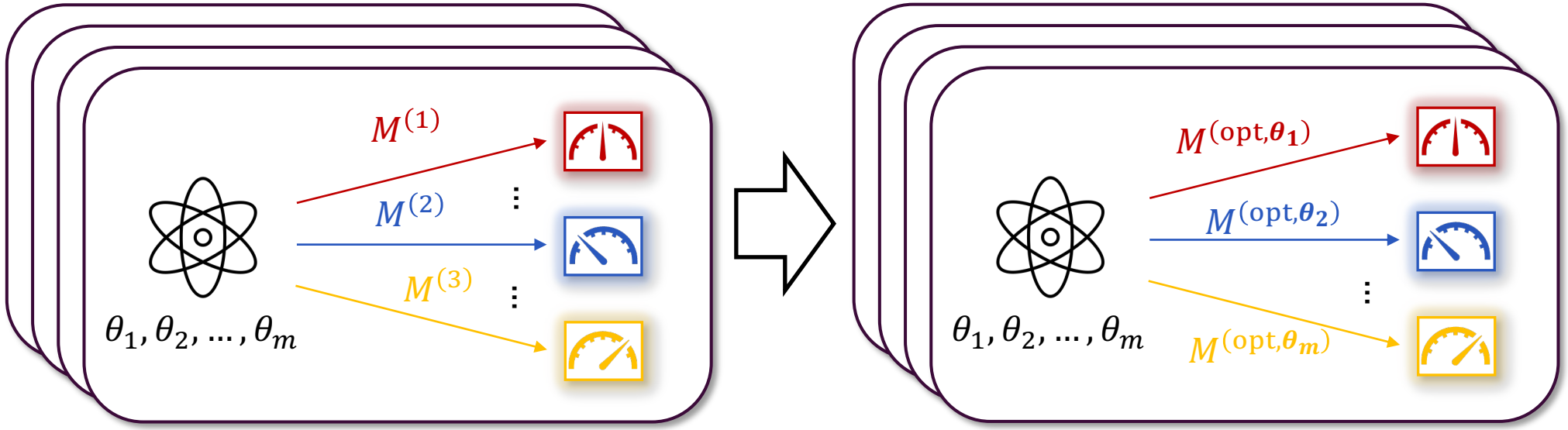
$$I(\rho_\theta, M)$$

Quantum Fisher Information:

$$\boldsymbol{J(\rho_\theta) = \max_{M} I(\rho_\theta, M)}$$

Braunstein & Caves PRL 1994

Multi-parameter Estimation



Fisher Information Matrix:

$$I_{ij}(\rho_\theta, M)$$

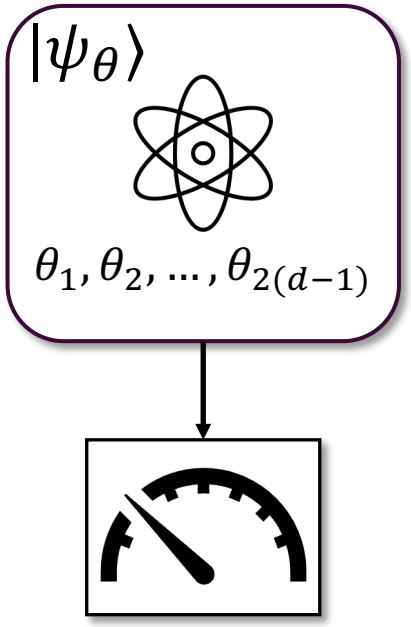
Quantum Fisher Information Matrix:

$$\boldsymbol{J(\rho_\theta) \succcurlyeq I(\rho_\theta, M)}$$

$$\boldsymbol{\nexists M,\ I_{ij}(\rho_\theta, M) = J_{ij}(\rho_\theta)} \text{ (usually)}$$

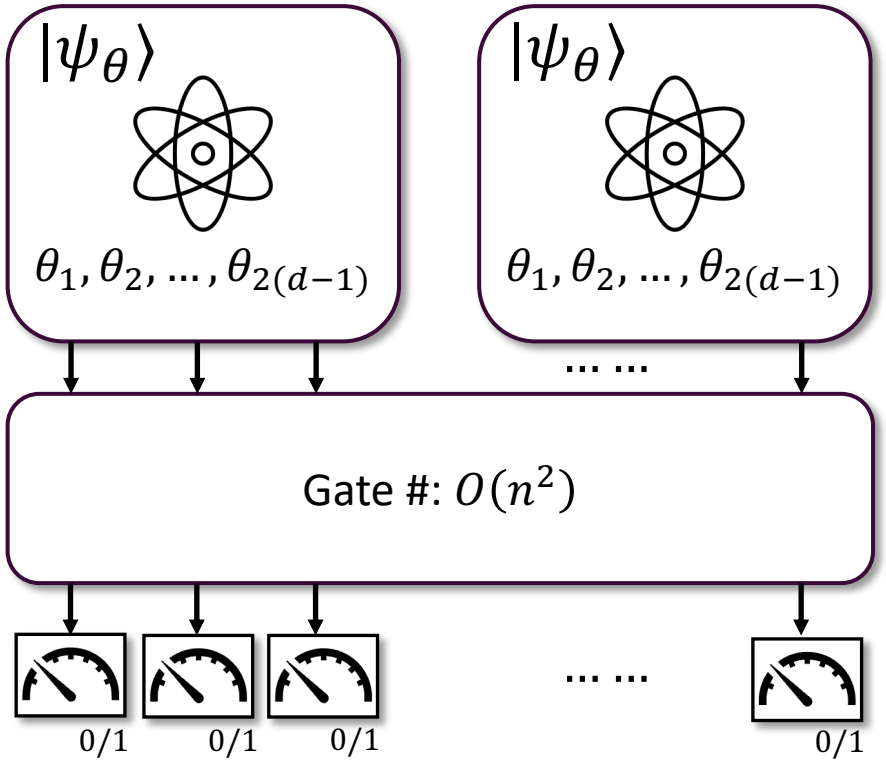
Pure States: Fisher-symmetric Measurement

Qubit number: n



$$I(\psi_\theta, M) = \frac{1}{2} J(\psi_\theta)$$

M: Complex measurement



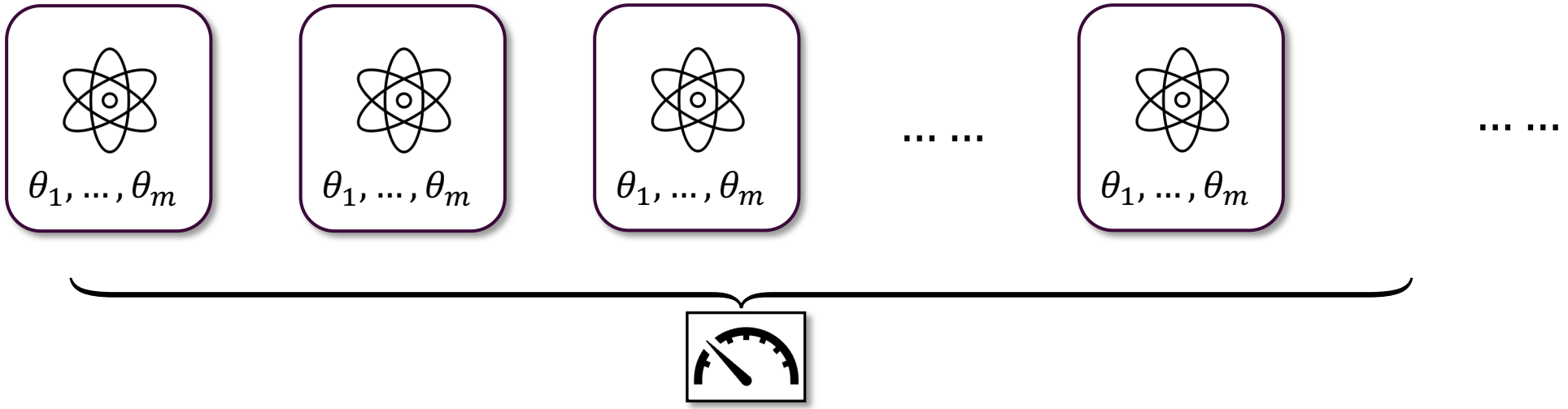
$$I(\psi_\theta^{\otimes 2}, M) = J(\psi_\theta^{\otimes 2})$$

M: Two-copy measurement

***Pure States**: Efficiently implementable individual measurement?*

Li et al. 2016
Zhu & Hayashi 2018

Collective Measurement: Holevo Cramér–Rao bound



Holevo Cramér–Rao bound (HCRB):

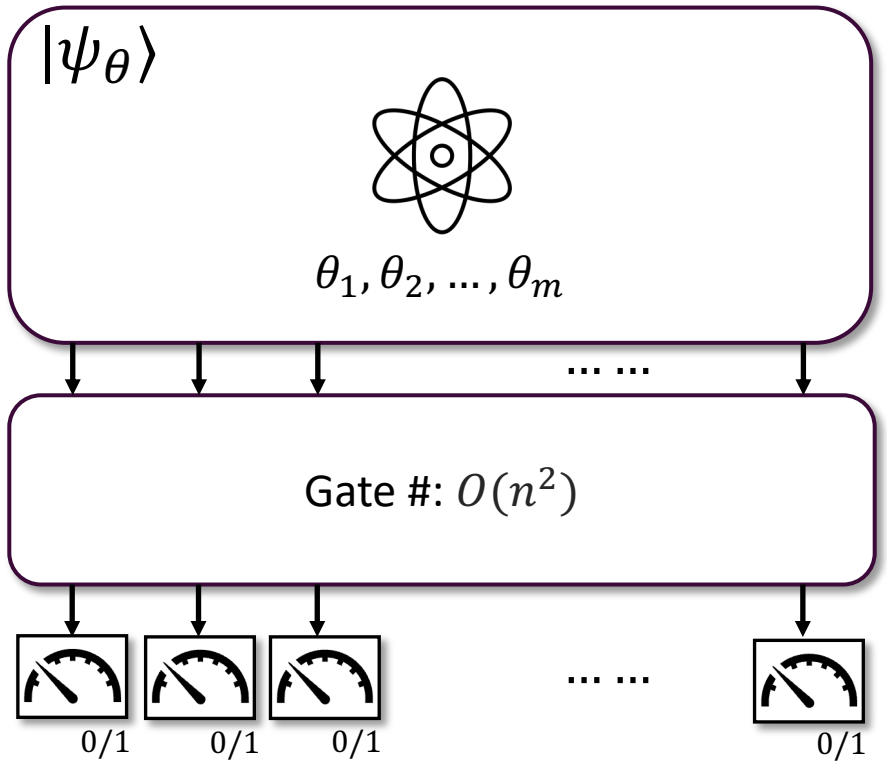
$$\mathrm{Tr}\big(W \cdot V(\theta)\big) \geq C_{\mathrm{H}}(\rho_\theta, W) = [\mathbf{1}, \mathbf{2}] \times \mathrm{Tr}(W \cdot J^{-1}(\rho_\theta))$$

- $\mathrm{Tr}\big(W \cdot V(\theta)\big)$: **averaged mean squared error** with weight matrix W.

***Mixed States**: How to implement the collective measurement?*

Gill & Guta 2013, Yamagata et al. 2013, Yang et al. 2019, Matsumoto 1997, Carollo et al. 2019, Tsang et al. 2020, .. etc.

Result I (Pure State + Efficient Individual Measurement)



Theorem: There exists some measurement M such that,

$$I(\psi_\theta, M) \geqslant \frac{1}{4} J(\psi_\theta),$$

for any pure state,

with finite-number outcomes,

and gate complexity $O(n^2)$.

SZ, Chen, PRXQuantum 2026

Proof Idea: **Metrology ⇐ Tomography**

Proof idea: Convert parameter estimation to learning quantum observables.

$$\boldsymbol{\theta_i - \theta_i^0} \approx \mathbf{Tr}\left(\boldsymbol{X_i}\left(\boldsymbol{\rho_\theta - \rho_{\theta^0}}\right)\right)$$

$$\hat{\theta}_i - \theta_i^0 \approx \sum_z \left(\hat{p}(z) - p_0(z)\right) x_z^{(i)}$$

$$\hat{p}(z) = \widehat{\mathrm{Tr}(\rho M_z)}$$
$$p_0(z) = \mathrm{Tr}(\rho_{\theta^0} M_z)$$

$$X_i = \sum_z x_z^{(i)} M_z$$

Parameter Estimation

Measurement Statistics

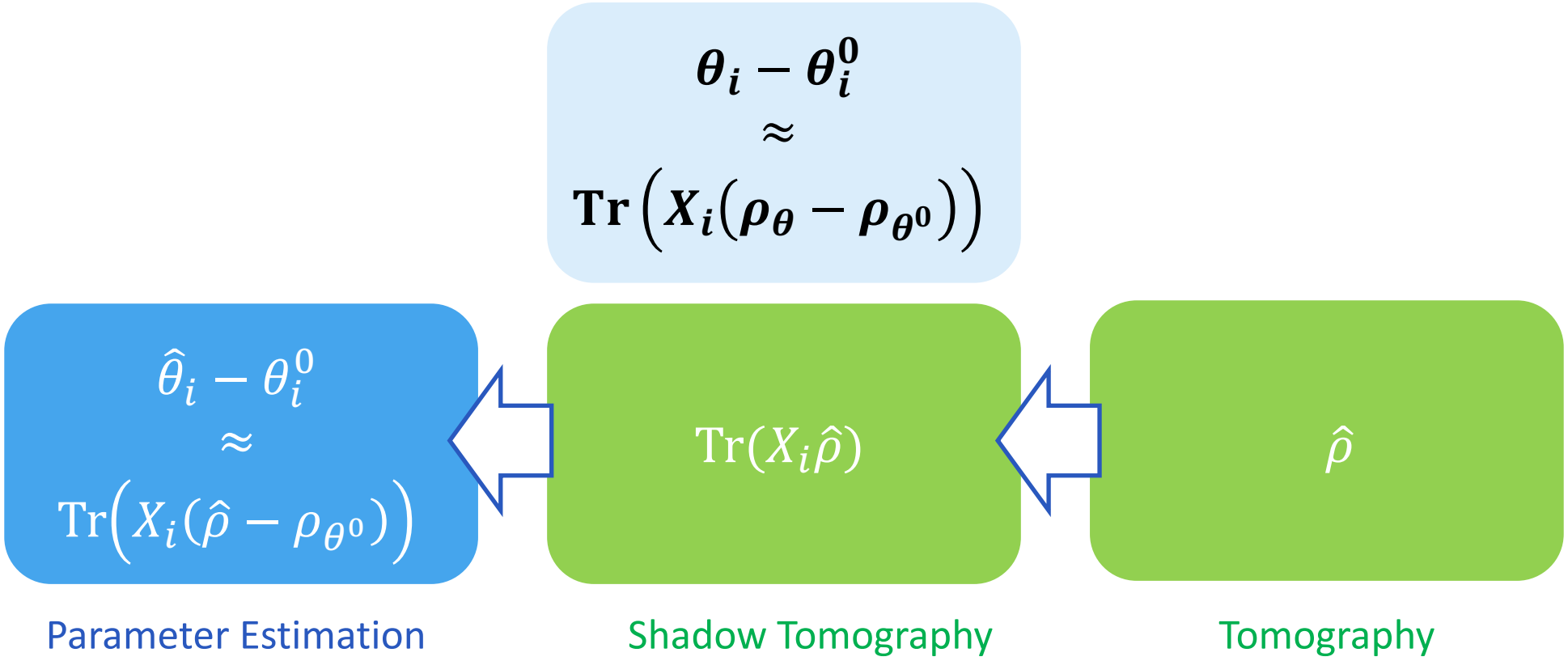
Measurement Basis M_z

When $[X_i, X_j] \neq 0$, it'll be difficult to find the optimal $\{M_z\}_z$

SZ, Chen, PRXQuantum 2026

Proof Idea: **Metrology ⇐ Tomography**

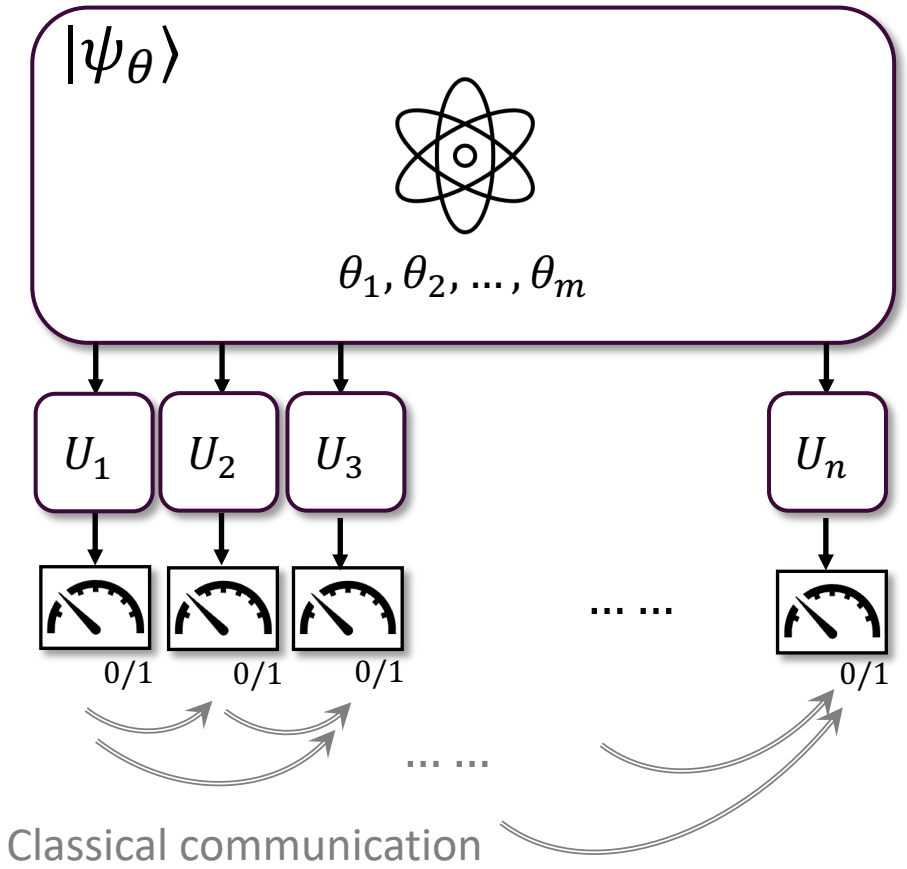
Proof idea: Convert parameter estimation to learning quantum observables.



Apply previous results on optimal shadow tomography to jointly estimate X_i with a high precision.

SZ, Chen, PRXQuantum 2026

Result II (Pure State + Single-qubit Measurement)



Theorem:

1. There exists a single-qubit, adaptive, measurement M, such that

$$I(\psi_\theta, M) \succcurlyeq \frac{1}{4n} J(\psi_\theta),$$

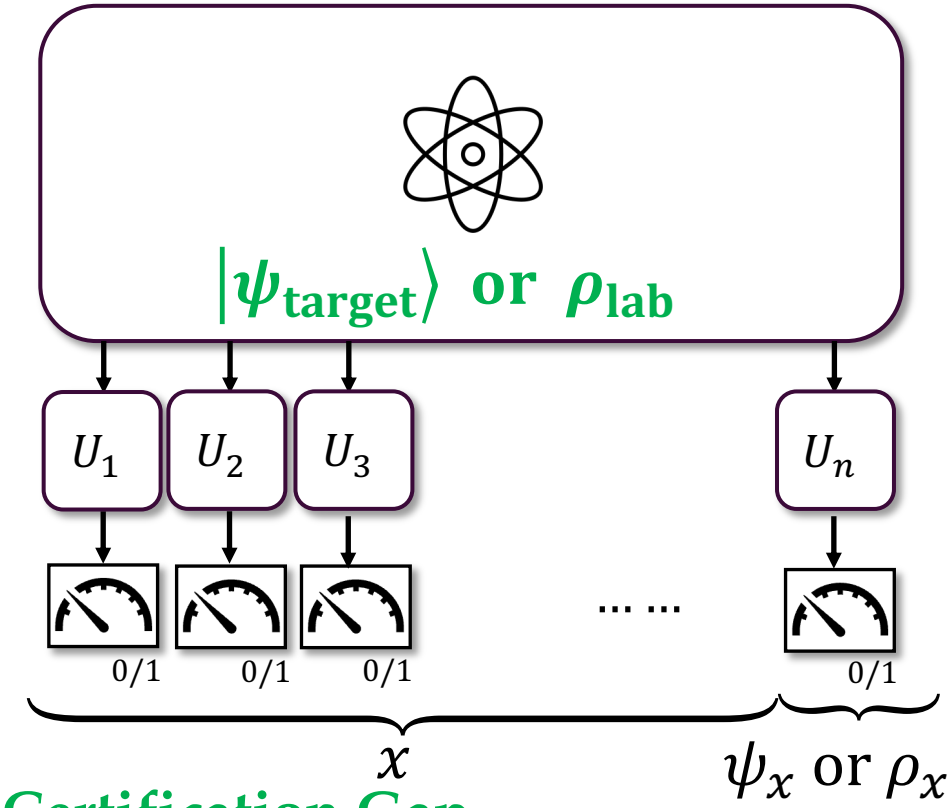
2. M is randomized Pauli measurement,

$$I(\psi_\theta, M) \succcurlyeq C \cdot J(\psi_\theta),$$

for most states ψ_θ.

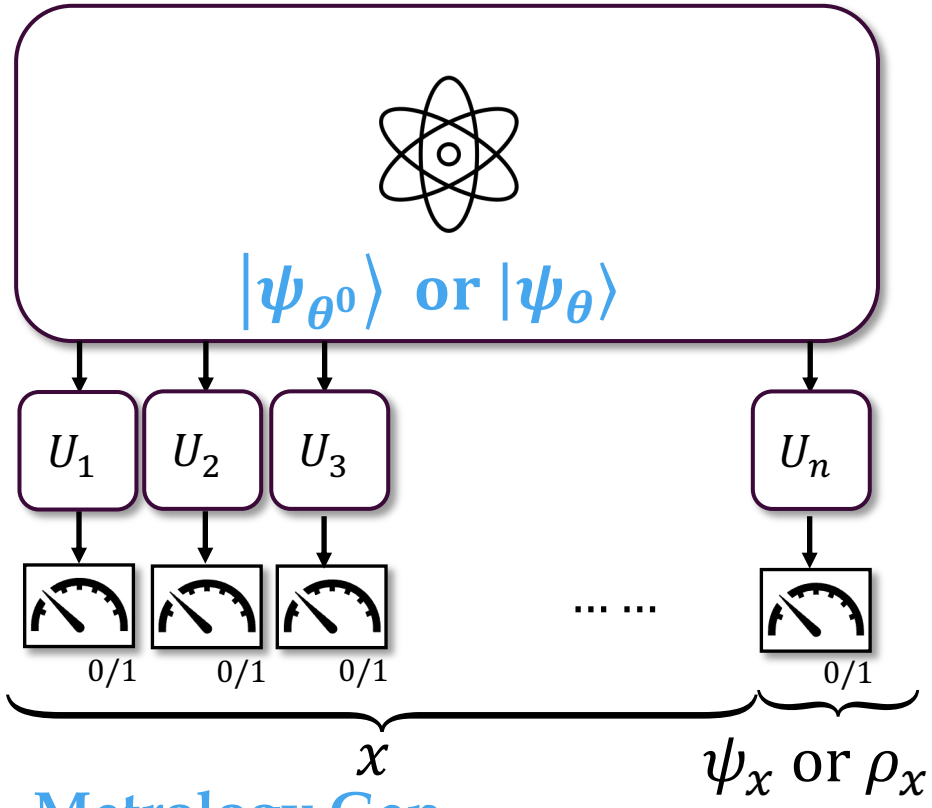
Mao, Chen, Huang, Preskill, **SZ**, 2026

Proof Idea: **Metrology ⇐ Certification**



Certification Gap

$$\mathbb{E}_x[d(\rho_x, \psi_x)] \geq \frac{d(\rho_{\text{lab}}, \psi_{\text{target}})}{\Delta}$$



Metrology Gap

$$I(\psi_\theta, M) \gtrsim \frac{1}{4\Delta} J(\psi_\theta)$$

Mao, Chen, Huang, Preskill, **SZ**, 2026 HPS 2025, GHO 2025, DLHLM 2026

Result III (Mixed State)

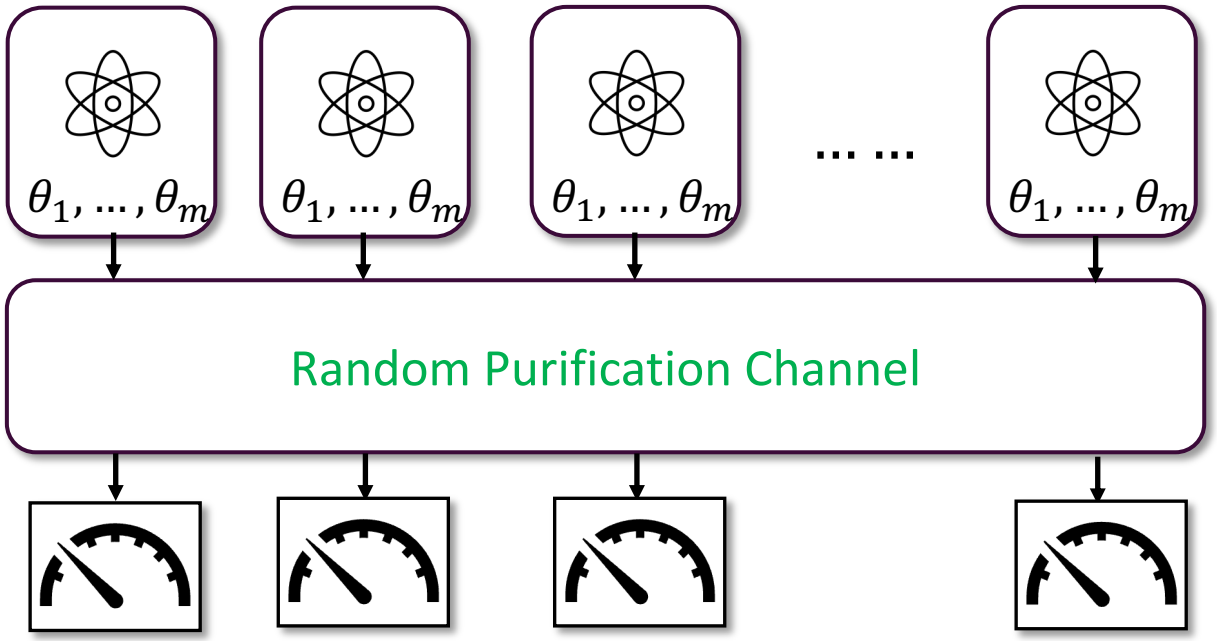
We construct an asymptotically unbiased estimator $\hat{\theta}$ s.t.

$$\lim_{N\to\infty} N \cdot \operatorname{Tr}\left(W \cdot V^{(N)}(\theta)\right) = C_{\mathrm{H}}(\rho_\theta, W)$$

Holevo Cramér–Rao bound

$$\left(\text{or } \lim_{n\to\infty} N\, V^{(N)}(\theta) = 2 \cdot J(\theta)^{-1}\right)$$

2* Quantum Cramér–Rao bound



Mixed States → Pure States

$$(\rho_\theta)_S \to (\psi_{\theta,\varphi})_{SE}$$

Random Purification Channel:

$$\mathbf{RPC}(\boldsymbol{\rho}^{\otimes N}) = \mathbb{E}_{\boldsymbol{U}}[\boldsymbol{\psi}_{\boldsymbol{U}}^{\otimes N}]$$

Circuit Complexity:

$$\mathrm{Poly}(N, \log d)$$

Summary of Part II

- Various measurement protocols were proposed to obtain the optimal or near-optimal sensitivity in multi-parameter quantum metrology, where **measurement incompatibility** is a key issue.
- These protocols are considered **efficiently implementable**, compared to previously known protocols.
- Proof ideas utilize connections between multi-parameter quantum metrology and **quantum learning**, including state tomography, state certification, purification techniques, etc.

Discussion

- Techniques from **quantum information processing** greatly enhances **quantum metrology** in
  - Protecting quantum signals from **quantum noise**
  - Resolving the **measurement incompatibility** issue in multi-parameter quantum metrology

Future directions:

- Develop new frameworks tailored for practical applications including specific quantum signals (e.g. time-dependent, non-Markovian signals), physical platforms, and application scenarios (e.g. with distinct target precision requirements, energy constraints, etc.).

Thank you!